MASS DEFECT AND BINDING ENERGY

The separate laws of Conservation of Mass and Conservation of Energy are not applied strictly on the nuclear level. It is possible to convert between mass and energy. Instead of two separate conservation laws, a single conservation law states that the sum of mass and energy is conserved. Mass does not magically appear and disappear at random. A decrease in mass will be accompanied by a corresponding increase in energy and vice versa.

EO 1.7 DEFINE the following terms:

a. Mass defect
b. Binding energy

EO 1.8 Given the atomic mass for a nuclide and the atomic masses of a neutron, proton, and electron, CALCULATE the mass defect and binding energy of the nuclide.

Mass Defect

Careful measurements have shown that the mass of a particular atom is always slightly less than the sum of the masses of the individual neutrons, protons, and electrons of which the atom consists. The difference between the mass of the atom and the sum of the masses of its parts is called the *mass defect* (Δm). The mass defect can be calculated using Equation (1-1). In calculating the mass defect it is important to use the full accuracy of mass measurements because the difference in mass is small compared to the mass of the atom. Rounding off the masses of atoms and particles to three or four significant digits prior to the calculation will result in a calculated mass defect of zero.

$$\Delta m = [\ Z(m_p + m_e) + (A-Z)m_n\ ] - m_{atom} \tag{1-1}$$

where:

Δm	=	mass defect (amu)
m_p	=	mass of a proton (1.007277 amu)
m_n	=	mass of a neutron (1.008665 amu)
m_e	=	mass of an electron (0.000548597 amu)
m_{atom}	=	mass of nuclide ${}^{A}_{Z}X$ (amu)
Z	=	atomic number (number of protons)
A	=	mass number (number of nucleons)

Example:

Calculate the mass defect for lithium-7. The mass of lithium-7 is 7.016003 amu.

Solution:

$$\Delta m = \left[Z\left(m_p + m_e\right) + \left(A - Z\right) m_n\right] - m_{atom}$$

$$\Delta m = \left[3\left(1.007826 \text{ amu}\right) + \left(7 - 3\right) 1.008665 \text{ amu}\right] - 7.016003 \text{ amu}$$

$$\Delta m = 0.0421335 \text{ amu}$$

Binding Energy

The loss in mass, or mass defect, is due to the conversion of mass to binding energy when the nucleus is formed. *Binding energy* is defined as the amount of energy that must be supplied to a nucleus to completely separate its nuclear particles (nucleons). It can also be understood as the amount of energy that would be released if the nucleus was formed from the separate particles. Binding energy is the energy equivalent of the mass defect. Since the mass defect was converted to binding energy (BE) when the nucleus was formed, it is possible to calculate the binding energy using a conversion factor derived by the mass-energy relationship from Einstein's Theory of Relativity.

Einstein's famous equation relating mass and energy is $E = mc^2$ where c is the velocity of light ($c = 2.998 \text{ x } 10^8$ m/sec). The energy equivalent of 1 amu can be determined by inserting this quantity of mass into Einstein's equation and applying conversion factors.

$$E = m\,c^2$$

$$= 1 \text{ amu}\left(\frac{1.6606 \text{ x } 10^{-27} \text{ kg}}{1 \text{ amu}}\right)\left(2.998 \text{ x } 10^8 \ \frac{\text{m}}{\text{sec}}\right)^2 \left(\frac{1 \text{ N}}{1 \ \frac{\text{kg-m}}{\text{sec}^2}}\right)\left(\frac{1 \text{ J}}{1 \text{ N-m}}\right)$$

$$= 1.4924 \text{ x } 10^{-10} \text{ J}\left(\frac{1 \text{ MeV}}{1.6022 \text{ x } 10^{-13} \text{ J}}\right)$$

$$= 931.5 \text{ MeV}$$

Conversion Factors:

1 amu	=	$1.6606 \text{ x } 10^{-27}$ kg
1 newton	=	1 kg-m/sec^2
1 joule	=	1 newton-meter
1 MeV	=	$1.6022 \text{ x } 10^{-13}$ joules

Since 1 amu is equivalent to 931.5 MeV of energy, the binding energy can be calculated using Equation (1-2).

$$\text{B.E.} = \Delta m \left(\frac{931.5 \ \text{MeV}}{1 \ \text{amu}} \right) \tag{1-2}$$

Example:

Calculate the mass defect and binding energy for uranium-235. One uranium-235 atom has a mass of 235.043924 amu.

Solution:

Step 1: Calculate the mass defect using Equation (1-1).

$$\Delta m = \left[Z \left(m_p + m_e \right) + \left(A - Z \right) m_n \right] - m_{atom}$$

$$\Delta m = \left[92 \left(1.007826 \ \text{amu} \right) + \left(235 - 92 \right) 1.008665 \ \text{amu} \right] - 235.043924 \ \text{amu}$$

$$\Delta m = 1.91517 \ \text{amu}$$

Step 2: Use the mass defect and Equation (1-2) to calculate the binding energy.

$$\text{B.E.} = \Delta m \left(\frac{931.5 \ \text{MeV}}{1 \ \text{amu}} \right)$$

$$= 1.91517 \ \text{amu} \left(\frac{931.5 \ \text{MeV}}{1 \ \text{amu}} \right)$$

$$= 1784 \ \text{MeV}$$

Energy Levels of Atoms

The electrons that circle the nucleus move in fairly well-defined orbits. Some of these electrons are more tightly bound in the atom than others. For example, only 7.38 eV is required to remove the outermost electron from a lead atom, while 88,000 eV is required to remove the innermost electron. The process of removing an electron from an atom is called ionization, and the energy required to remove the electron is called the ionization energy.

In a neutral atom (number of electrons = Z) it is possible for the electrons to be in a variety of different orbits, each with a different energy level. The state of lowest energy is the one in which the atom is normally found and is called the ground state. When the atom possesses more energy than its ground state energy, it is said to be in an excited state.

An atom cannot stay in the excited state for an indefinite period of time. An excited atom will eventually transition to either a lower-energy excited state, or directly to its ground state, by emitting a discrete bundle of electromagnetic energy called an x-ray. The energy of the x-ray will be equal to the difference between the energy levels of the atom and will typically range from several eV to 100,000 eV in magnitude.

Energy Levels of the Nucleus

The nucleons in the nucleus of an atom, like the electrons that circle the nucleus, exist in shells that correspond to energy states. The energy shells of the nucleus are less defined and less understood than those of the electrons. There is a state of lowest energy (the ground state) and discrete possible excited states for a nucleus. Where the discrete energy states for the electrons of an atom are measured in eV or keV, the energy levels of the nucleus are considerably greater and typically measured in MeV.

A nucleus that is in the excited state will not remain at that energy level for an indefinite period. Like the electrons in an excited atom, the nucleons in an excited nucleus will transition towards their lowest energy configuration and in doing so emit a discrete bundle of electromagnetic radiation called a gamma ray (γ-ray). The only differences between x-rays and γ-rays are their energy levels and whether they are emitted from the electron shell or from the nucleus.

The ground state and the excited states of a nucleus can be depicted in a nuclear energy-level diagram. The nuclear energy-level diagram consists of a stack of horizontal bars, one bar for each of the excited states of the nucleus. The vertical distance between the bar representing an excited state and the bar representing the ground state is proportional to the energy level of the excited state with respect to the ground state. This difference in energy between the ground state and the excited state is called the excitation energy of the excited state. The ground state of a nuclide has zero excitation energy. The bars for the excited states are labeled with their respective energy levels. Figure 7 is the energy level diagram for nickel-60.

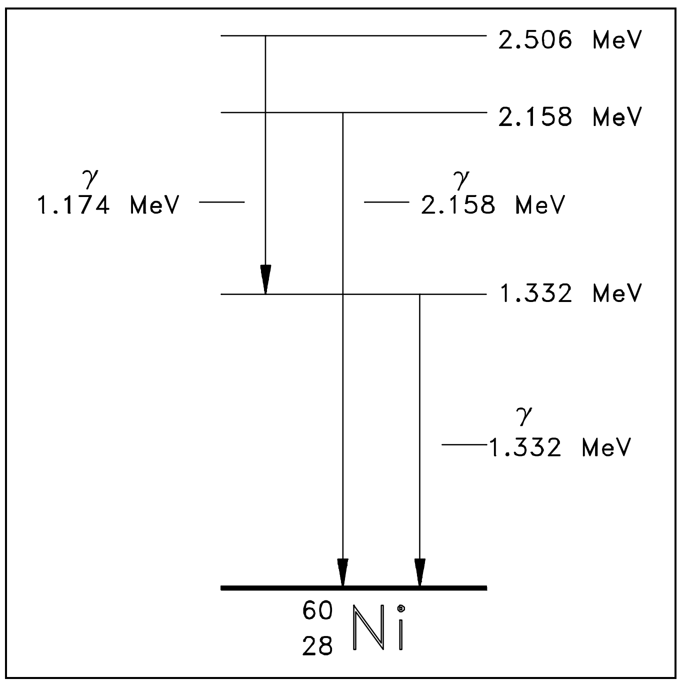

Figure 7 Energy Level Diagram - Nickel-60

Summary

The important information in this chapter is summarized below.

Mass Defect and Binding Energy Summary

- Mass defect is the difference between the mass of the atom and the sum of the masses of its constituent parts.
- Binding energy is the amount of energy that must be supplied to a nucleus to completely separate its nuclear particles. Binding energy is the energy equivalent of the mass defect.
- Mass defect can be calculated by using the equation below.

$$\Delta m = [\ Z(m_p + m_e) + (A\text{-}Z)m_n\ ] - m_{atom}$$

- Binding energy can be calculated by multiplying the mass defect by the factor of 931.5 MeV per amu.