RADIOACTIVITY

The rate at which a sample of radioactive material decays is not constant. As individual atoms of the material decay, there are fewer of those types of atoms remaining. Since the rate of decay is directly proportional to the number of atoms, the rate of decay will decrease as the number of atoms decreases.

EO 2.5 DEFINE the following terms:

- **a. Radioactivity**
- **b. Curie**
- **c. Becquerel**
- **d. Radioactive decay constant**
- **e. Radioactive half-life**

EO 2.6 Given the number of atoms and either the half-life or decay constant of a nuclide, CALCULATE the activity.

EO 2.7 Given the initial activity and the decay constant of a nuclide, CALCULATE the activity at any later time.

EO 2.8 CONVERT between the half-life and decay constant for a nuclide.

EO 2.9 Given the Chart of the Nuclides and the original activity, PLOT the radioactive decay curve for a nuclide on either linear or semi-log coordinates.

EO 2.10 DEFINE the following terms:

- **a. Radioactive equilibrium**
- **b. Transient radioactive equilibrium**

Radioactive Decay Rates

Radioactivity is the property of certain nuclides of spontaneously emitting particles or gamma radiation. The decay of radioactive nuclides occurs in a random manner, and the precise time at which a single nucleus will decay cannot be determined. However, the average behavior of a very large sample can be predicted accurately by using statistical methods. These studies have revealed that there is a certain probability that in a given time interval a certain fraction of the nuclei within a sample of a particular nuclide will decay. This probability per unit time that an atom of a nuclide will decay is known as the *radioactive decay constant*, λ. The units for the decay constant are inverse time such as 1/second, 1/minute, 1/hour, or 1/year. These decay constant units can also be expressed as $second^{-1}$, $minute^{-1}$, $hour^{-1}$, and $year^{-1}$.

The *activity* (A) of a sample is the rate of decay of that sample. This rate of decay is usually measured in the number of disintegrations that occur per second. For a sample containing millions of atoms, the activity is the product of the decay constant and the number of atoms present in the sample.

The relationship between the activity, number of atoms, and decay constant is shown in Equation (1-3).

$$A = \lambda N \tag{1-3}$$

where:

A = Activity of the nuclide (disintegrations/second)
λ = decay constant of the nuclide (second^{-1})
N = Number of atoms of the nuclide in the sample

Since λ is a constant, the activity and the number of atoms are always proportional.

Units of Measurement for Radioactivity

Two common units to measure the activity of a substance are the curie (Ci) and becquerel (Bq). A *curie* is a unit of measure of the rate of radioactive decay equal to 3.7×10^{10} disintegrations per second. This is approximately equivalent to the number of disintegrations that one gram of radium-226 will undergo in one second. A *becquerel* is a more fundamental unit of measure of radioactive decay that is equal to 1 disintegration per second. Currently, the curie is more widely used in the United States, but usage of the becquerel can be expected to broaden as the metric system slowly comes into wider use. The conversion between curies and becquerels is shown below.

1 curie = 3.7×10^{10} becquerels

Variation of Radioactivity Over Time

The rate at which a given radionuclide sample decays is stated in Equation (1-3) as being equal to the product of the number of atoms and the decay constant. From this basic relationship it is possible to use calculus to derive an expression which can be used to calculate how the number of atoms present will change over time. The derivation is beyond the scope of this text, but Equation (1-4) is the useful result.

$$N = N_o e^{-\lambda t} \tag{1-4}$$

where:

N = number of atoms present at time t
N_o = number of atoms initially present
λ = decay constant (time^{-1})
t = time

Since the activity and the number of atoms are always proportional, they may be used interchangeably to describe any given radionuclide population. Therefore, the following is true.

$$A = A_o \, e^{-\lambda t} \tag{1-5}$$

where:

A	=	activity present at time t
A_o	=	activity initially present
λ	=	decay constant ($time^{-1}$)
t	=	time

Radioactive Half-Life

One of the most useful terms for estimating how quickly a nuclide will decay is the radioactive half-life. The *radioactive half-life* is defined as the amount of time required for the activity to decrease to one-half of its original value. A relationship between the half-life and decay constant can be developed from Equation (1-5). The half-life can be calculated by solving Equation (1-5) for the time, t, when the current activity, A, equals one-half the initial activity A_o.

First, solve Equation (1-5) for t.

$$A = A_o \, e^{-\lambda t}$$

$$\frac{A}{A_o} = e^{-\lambda t}$$

$$\ln\left(\frac{A}{A_o}\right) = -\lambda \, t$$

$$t = \frac{-\ln\left(\frac{A}{A_o}\right)}{\lambda}$$

If A is equal to one-half of A_o, then A/A_o is equal to one-half. Substituting this in the equation above yields an expression for $t_{1/2}$.

$$t_{1/2} = \frac{-\ln\left(\frac{1}{2}\right)}{\lambda} \tag{1-6}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

The basic features of decay of a radionuclide sample are shown by the graph in Figure 10.

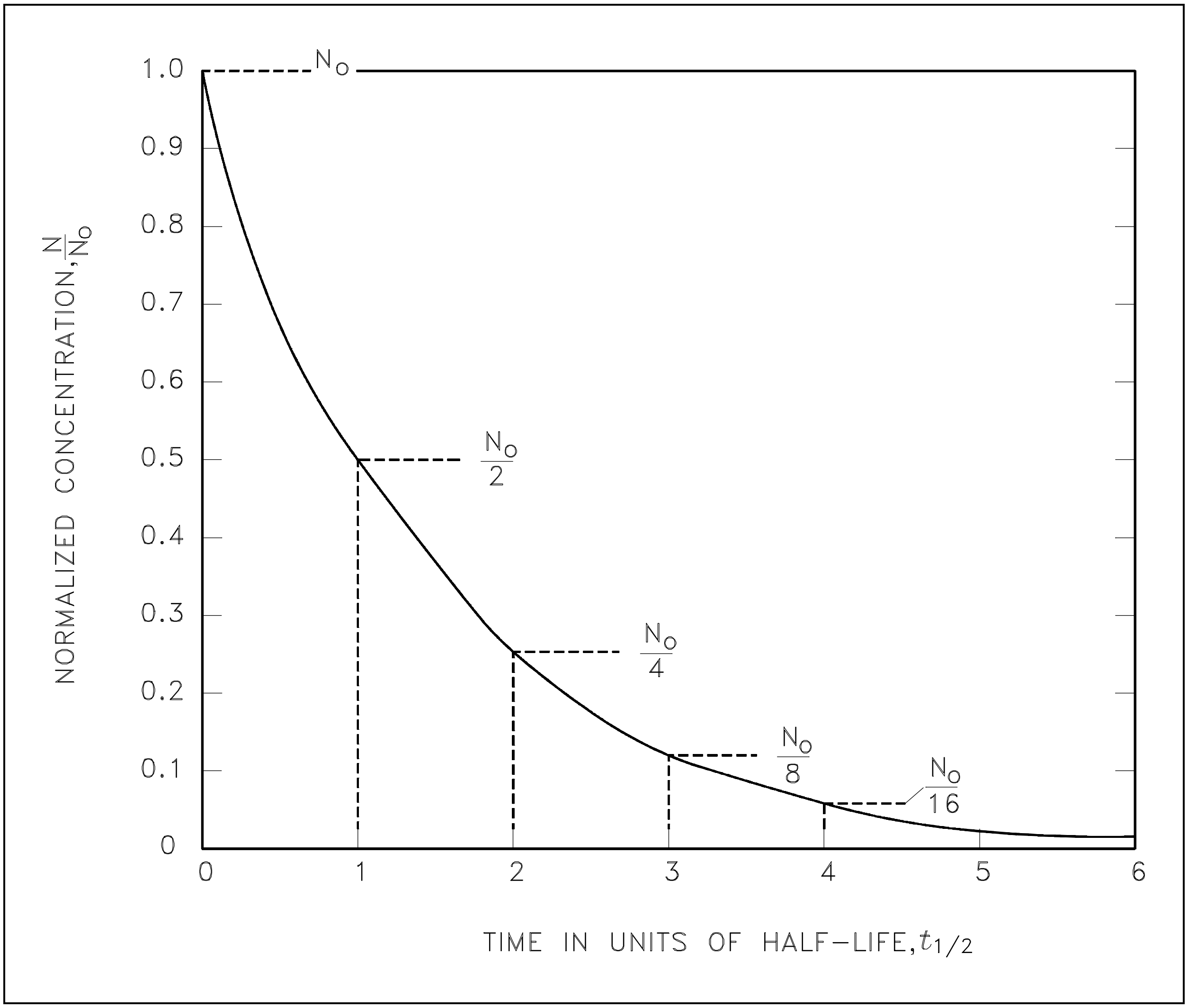

Figure 10 Radioactive Decay as a Function of Time in Units of Half-Life

Assuming an initial number of atoms N_o, the population, and consequently, the activity may be noted to decrease by one-half of this value in a time of one half-life. Additional decreases occur so that whenever one half-life elapses, the number of atoms drops to one-half of what its value was at the beginning of that time interval. After five half-lives have elapsed, only 1/32, or 3.1%, of the original number of atoms remains. After seven half-lives, only 1/128, or 0.78%, of the atoms remains. The number of atoms existing after 5 to 7 half-lives can usually be assumed to be negligible. The Chemistry Fundamentals Handbook contains additional information on calculating the number of atoms contained within a sample.

Example:

A sample of material contains 20 micrograms of californium-252.

Californium-252 has a half-life of 2.638 years.

Calculate:

(a) The number of californium-252 atoms initially present
(b) The activity of the californium-252 in curies
(c) The number of californium-252 atoms that will remain in 12 years
(d) The time it will take for the activity to reach 0.001 curies

Solution:

(a) The number of atoms of californium-252 can be determined as below.

$$N_{Cf-252} = \text{mass} \left(\frac{1 \text{ mole}}{\text{isotopic mass}} \right) \left(\frac{N_A}{1 \text{ mole}} \right)$$

$$= (20 \text{ x } 10^{-6} \text{ g}) \left(\frac{1 \text{ mole}}{252.08 \text{ g}} \right) \left(\frac{6.022 \text{ x } 10^{23} \text{ atoms}}{1 \text{ mole}} \right)$$

$$= 4.78 \text{ x } 10^{16} \text{ atoms}$$

(b) First, use Equation (1-6) to calculate the decay constant.

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$= \frac{0.693}{2.638 \text{ years}}$$

$$= 0.263 \text{ year}^{-1}$$

Use this value for the decay constant in Equation (1-3) to determine the activity.

$$A = \lambda N$$

$$= (0.263 \text{ year}^{-1})(4.78 \text{ x } 10^{16} \text{ atoms}) \left(\frac{1 \text{ year}}{365.25 \text{ days}} \right) \left(\frac{1 \text{ day}}{24 \text{ hours}} \right) \left(\frac{1 \text{ hour}}{3600 \text{ seconds}} \right)$$

$$= \left(3.98 \text{ x } 10^{8} \frac{\text{disintegrations}}{\text{second}} \right) \left(\frac{1 \text{ curie}}{3.7 \text{ x } 10^{10} \text{ disintegrations/sec}} \right)$$

$$= 0.0108 \text{ curies}$$

(c) The number of californium atoms that will remain in 12 years can be calculated from Equation (1-4).

$$N = N_o \, e^{-\lambda t}$$

$$= \left(4.78 \text{ x } 10^{16}\right) e^{-(0.263/\text{yr})(12 \text{ yr})}$$

$$= 2.04 \text{ x } 10^{15}$$

(d) The time that it will take for the activity to reach 0.001 Ci can be determined from Equation (1-5). First, solve Equation (1-5) for time.

$$A = A_o \, e^{-\lambda t}$$

$$\frac{A}{A_o} = e^{-\lambda t}$$

$$\ln\left(\frac{A}{A_o}\right) = -\lambda t$$

$$t = \frac{-\ln\left(\frac{A}{A_o}\right)}{\lambda}$$

Inserting the appropriate values in the right side of this equation will result in the required time.

$$t = \frac{-\ln\left(\frac{0.001 \text{ Ci}}{0.0108 \text{ Ci}}\right)}{0.263 \text{ year}^{-1}}$$

$$t = 9.05 \text{ years}$$

Plotting Radioactive Decay

It is useful to plot the activity of a nuclide as it changes over time. Plots of this type can be used to determine when the activity will fall below a certain level. This plot is usually done showing activity on either a linear or a logarithmic scale. The decay of the activity of a single nuclide on a logarithmic scale will plot as a straight line because the decay is exponential.

Example:

Plot the radioactive decay curve for nitrogen-16 over a period of 100 seconds. The initial activity is 142 curies and the half-life of nitrogen-16 is 7.13 seconds. Plot the curve on both linear rectangular coordinates and on a semi-log scale.

Solution:

First, use Equation (1-6) to calculate the decay constant corresponding to a half-life of 7.13 seconds.

$$t_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$\lambda = \frac{0.693}{7.13 \ \text{seconds}}$$

$$\lambda = 0.0972 \ \text{second}^{-1}$$

Use the decay constant determined above to calculate the activity at various times using Equation (1-5).

$$A = A_o \ e^{-\lambda t}$$

Time	Activity
0 seconds	142 Ci
20 seconds	20.3 Ci
40 seconds	2.91 Ci
60 seconds	0.416 Ci
80 seconds	0.0596 Ci
100 seconds	0.00853 Ci

Plotting the data points calculated above on both linear and semilog scales results in the graphs shown in Figure 11.

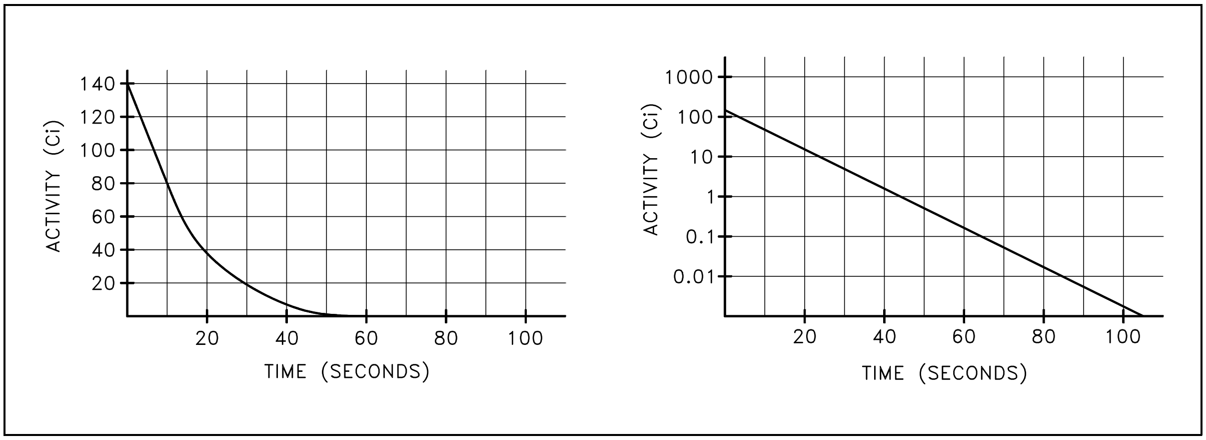

Figure 11 Linear and Semi-log Plots of Nitrogen-16 Decay

If a substance contains more than one radioactive nuclide, the total activity is the sum of the individual activities of each nuclide. As an example, consider a sample of material that contained 1×10^{6} atoms of iron-59 that has a half-life of 44.51 days ($\lambda = 1.80 \times 10^{7}$ sec^{-1}), 1×10^{6} atoms of manganese-54 that has a half-life of 312.2 days ($\lambda = 2.57 \times 10^{-8}$ sec^{-1}), and 1×10^{6} atoms of cobalt-60 that has a half-life of 1925 days ($\lambda = 4.17 \times 10^{-9}$ sec^{-1}).

The initial activity of each of the nuclides would be the product of the number of atoms and the decay constant.

$$A_{Fe-59} = N_{Fe-59} \, \lambda_{Fe-59}$$
$$= (1 \ \text{x} \ 10^{6} \ \text{atoms}) \, (1.80 \ \text{x} \ 10^{-7} \ \text{sec}^{-1})$$
$$= 0.180 \ \text{Ci}$$

$$A_{Mn-54} = N_{Mn-54} \, \lambda_{Mn-54}$$
$$= (1 \ \text{x} \ 10^{6} \ \text{atoms}) \, (2.57 \ \text{x} \ 10^{-8} \ \text{sec}^{-1})$$
$$= 0.0257 \ \text{Ci}$$

$$A_{Co-60} = N_{Co-60} \, \lambda_{Co-60}$$
$$= (1 \ \text{x} \ 10^{6} \ \text{atoms}) \, (4.17 \ \text{x} \ 10^{-9} \ \text{sec}^{-1})$$
$$= 0.00417 \ \text{Ci}$$

Plotting the manner in which the activities of each of the three nuclides decay over time demonstrates that initially the activity of the shortest-lived nuclide (iron-59) dominates the total activity, then manganese-54 dominates. After almost all of the iron and manganese have decayed away, the only contributor to activity will be the cobalt-60. A plot of this combined decay is shown in Figure 12.

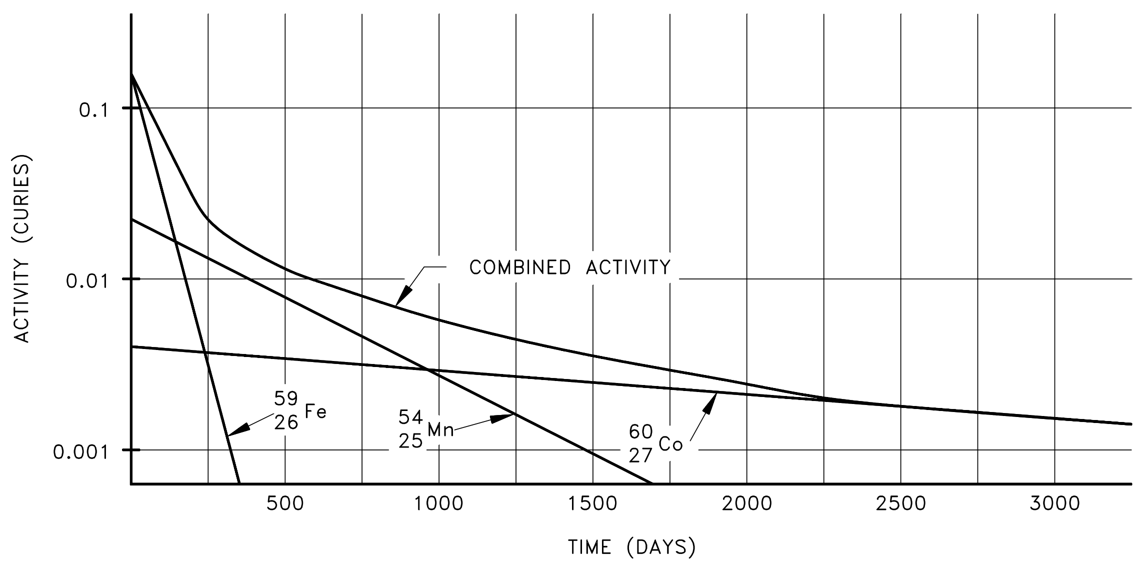

Figure 12 Combined Decay of Iron-56, Manganese-54, and Cobalt-60

Radioactive Equilibrium

Radioactive equilibrium exists when a radioactive nuclide is decaying at the same rate at which it is being produced. Since the production rate and decay rate are equal, the number of atoms present remains constant over time.

An example of radioactive equilibrium is the concentration of sodium-24 in the coolant circulating through a sodium-cooled nuclear reactor. Assume that the sodium-24 is being produced at a rate of 1×10^6 atoms per second. If the sodium-24 were stable and did not decay, the amount of sodium-24 present after some period of time could be calculated by multiplying the production rate by the amount of time. Plotting the amount of material present would result in the graph in Figure 13.

However, sodium-24 is not stable, and it decays with a half-life of 14.96 hours. If no sodium-24 is present initially and production starts at a rate of 1×10^6 atoms per second, the rate of decay will initially be zero because there is no sodium-24 present to decay. The rate of decay of sodium-24 will increase as the amount of sodium-24 increases.

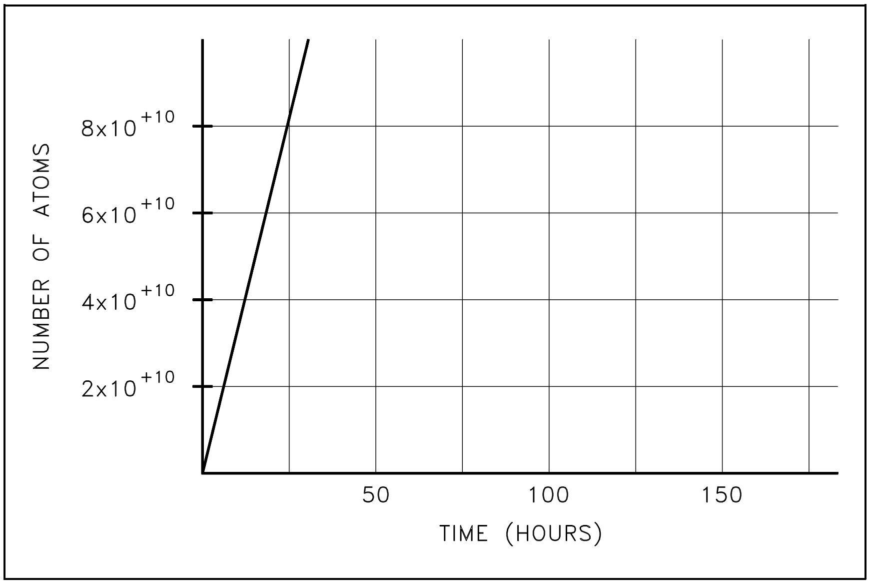

Figure 13 Cumulative Production of Sodium-24 Over Time

The amount of sodium-24 present will initially increase rapidly, then it will increase at a continually decreasing rate until the rate of decay is equal to the rate of production. It is possible to calculate how much sodium-24 will be present at equilibrium by setting the production rate (R) equal to the decay rate (λ N).

$$R = \lambda N$$

$$N = \frac{R}{\lambda}$$

where:

R = production rate (atoms/second)
λ = decay constant (second^{-1})
N = number of atoms

It is possible to calculate the equilibrium value for sodium-24 being produced at a rate of 1×10^6 atoms/second.

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$= \frac{0.693}{14.96 \text{ hours}} \left(\frac{1 \text{ hour}}{3600 \text{ seconds}} \right)$$

$$= 1.287 \times 10^{-5} \text{ second}^{-1}$$

$$N = \frac{R}{\lambda}$$

$$= \frac{1 \times 10^6 \ \frac{\text{atoms}}{\text{second}}}{1.287 \times 10^{-5} \text{ second}^{-1}}$$

$$= 7.77 \times 10^{10} \text{ atoms}$$

The development of the equation to calculate how the amount of sodium-24 changes over time as it approaches the equilibrium value is beyond the scope of this handbook. However, the equation is presented below.

$$N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$$

This equation can be used to calculate the values of the amount of sodium-24 present at different times. As the time increases, the exponential term approaches zero, and the number of atoms present will approach R/λ. A plot of the approach of sodium-24 to equilibrium is shown in Figure 14.

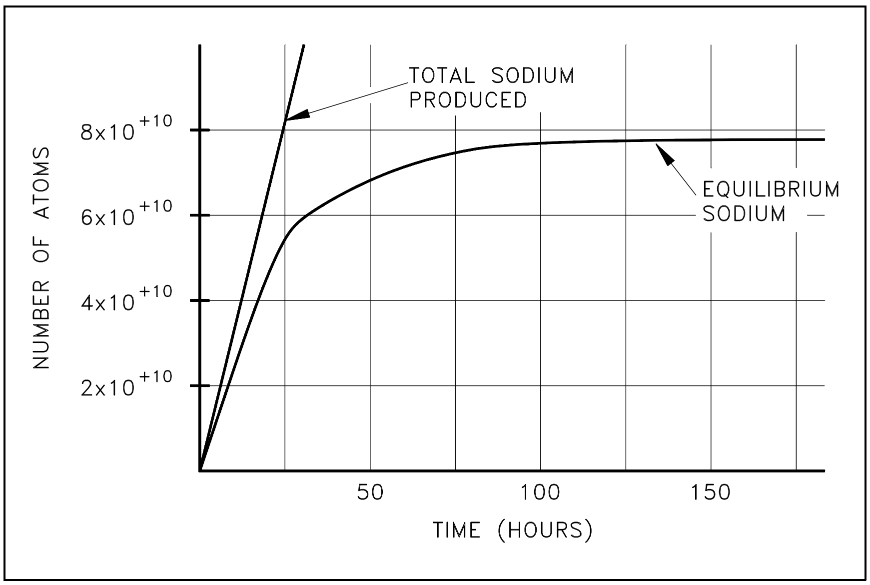

Figure 14 Approach of Sodium-24 to Equilibrium

Transient Radioactive Equilibrium

Transient radioactive equilibrium occurs when the parent nuclide and the daughter nuclide decay at essentially the same rate.

For transient equilibrium to occur, the parent must have a long half-life when compared to the daughter. An example of this type of compound decay process is barium-140, which decays by beta emission to lanthanum-140, which in turn decays by beta emission to stable cerium-140.

$$^{140}_{56}Ba \xrightarrow[12.75\ days]{\beta^-} {}^{140}_{57}La \xrightarrow[1.678\ days]{\beta^-} {}^{140}_{58}Ce$$

The decay constant for barium-140 is considerably smaller than the decay constant for lanthanum-140. Remember that the rate of decay of both the parent and daughter can be represented as λN. Although the decay constant for barium-140 is smaller, the actual rate of decay (λN) is initially larger than that of lanthanum-140 because of the great difference in their initial concentrations. As the concentration of the daughter increases, the rate of decay of the daughter will approach and eventually match the decay rate of the parent. When this occurs, they are said to be in transient equilibrium. A plot of the barium-lanthanum-cerium decay chain reaching transient equilibrium is shown in Figure 15.

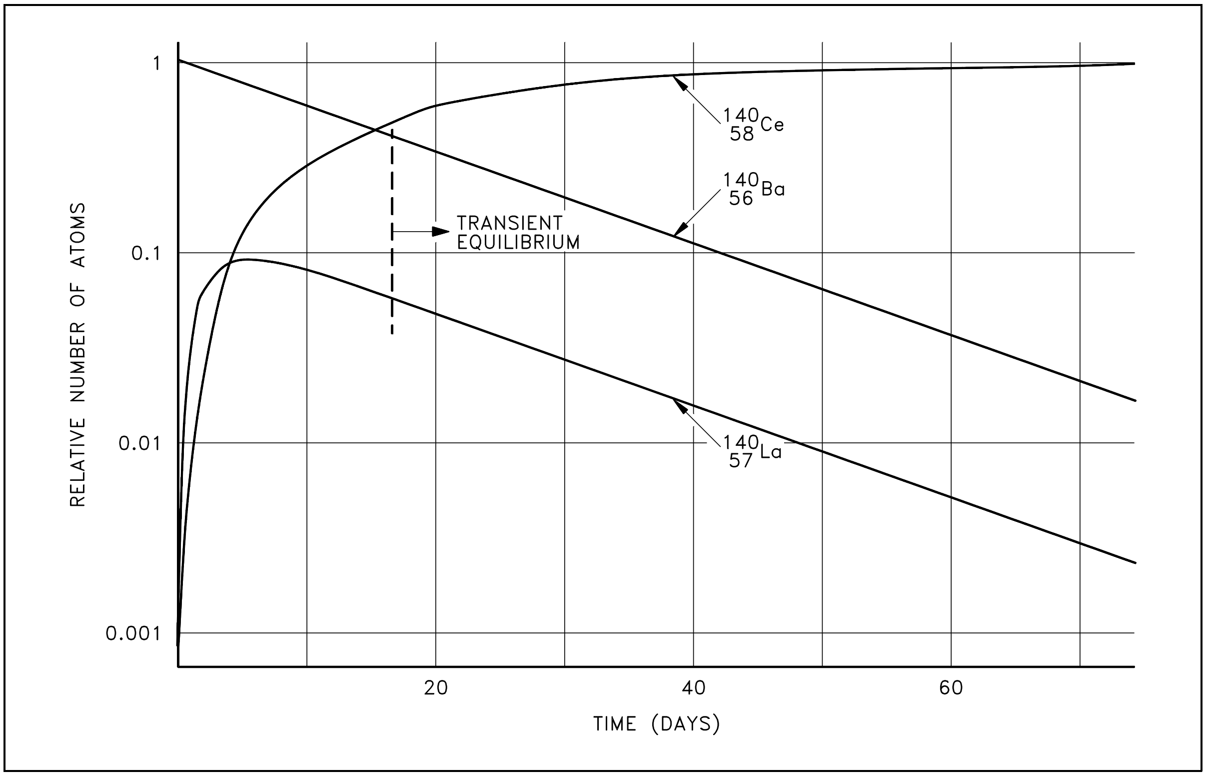

Figure 15 Transient Equilibrium in the Decay of Barium-140

Secular equilibrium occurs when the parent has an extremely long half-life. In the long decay chain for a naturally radioactive element, such as thorium-232, where all of the elements in the chain are in secular equilibrium, each of the descendants has built up to an equilibrium amount and all decay at the rate set by the original parent. The only exception is the final stable element on the end of the chain. Its number of atoms is constantly increasing.

Summary

The important information in this chapter is summarized on the following page.

Radioactivity Summary

- Radioactivity is the decay of unstable atoms by the emission of particles and electromagnetic radiation.
- A curie (Ci) is a unit of radioactivity equal to 3.7×10^{10} disintegrations per second.
- A becquerel (Bq) is a unit of radioactivity equal to 1 disintegration per second.
- The radioactive decay constant (λ) is the probability per unit time that an atom will decay.
- The radioactive half-life is the amount of time required for the activity to decrease to one-half its original value.
- The activity of a substance can be calculated from the number of atoms and the decay constant based on the equation below.

$$A = \lambda N$$

- The amount of activity remaining after a particular time can be calculated from the equation below.

$$A = A_o e^{-\lambda t}$$

- The relationship between the decay constant and the half-life is shown below.

$$t_{1/2} = \frac{0.693}{\lambda}$$

- Plots of radioactive decay can be useful to describe the variation of activity over time. If decay is plotted using semi-log scale the plot results in a straight line.
- Radioactive equilibrium exists when the production rate of a material is equal to the removal rate.
- Transient radioactive equilibrium exists when the parent nuclide and the daughter nuclide decay at essentially the same rate. This occurs only when the parent has a long half-life compared to the daughter.