NUCLEAR CROSS SECTIONS AND NEUTRON FLUX

To determine the frequency of neutron interactions, it is necessary to describe the availability of neutrons to cause interaction and the probability of a neutron interacting with material. The availability of neutrons and the probability of interaction are quantified by the neutron flux and nuclear cross section.

EO 2.1 DEFINE the following terms:

a.	**Atom density**	**d.**	**Barn**
b.	**Neutron flux**	**e.**	**Macroscopic cross section**
c.	**Microscopic cross section**	**f.**	**Mean free path**

EO 2.2 EXPRESS macroscopic cross section in terms of microscopic cross section.

EO 2.3 DESCRIBE how the absorption cross section of typical nuclides varies with neutron energy at energies below the resonance absorption region.

EO 2.4 DESCRIBE the cause of resonance absorption in terms of nuclear energy levels.

EO 2.5 DESCRIBE the energy dependence of resonance absorption peaks for typical light and heavy nuclei.

EO 2.6 EXPRESS mean free path in terms of macroscopic cross section.

EO 2.7 Given the number densities (or total density and component fractions) and microscopic cross sections of components, CALCULATE the macroscopic cross section for a mixture.

EO 2.8 CALCULATE a macroscopic cross section given a material density, atomic mass, and microscopic cross section.

EO 2.9 EXPLAIN neutron shadowing or self-shielding.

Introduction

Fission neutrons are born with an average energy of about 2 MeV. These fast neutrons interact with the reactor core materials in various absorption and scattering reactions. Collisions that result in scattering are useful in slowing neutrons to thermal energies. Thermal neutrons may be absorbed by fissile nuclei to produce more fissions or be absorbed in fertile material for conversion to fissionable fuel. Absorption of neutrons in structural components, coolant, and other non-fuel material results in the removal of neutrons without fulfilling any useful purpose.

To safely and efficiently operate a nuclear reactor it is necessary to predict the probability that a particular absorption or scattering reaction will occur. Once these probabilities are known, if the availability of neutrons can be determined, then the rate at which these nuclear reactions take place can be predicted.

Atom Density

One important property of a material is the atom density. The *atom density* is the number of atoms of a given type per unit volume of the material. To calculate the atom density of a substance use Equation (2-1).

$$N = \frac{\rho \, N_A}{M} \qquad (2\text{-}1)$$

where:

N = atom density (atoms/cm^3)
ρ = density (g/cm^3)
N_A = Avogadro's number (6.022×10^{23} atoms/mole)
M = gram atomic weight

Example:

A block of aluminum has a density of 2.699 g/cm^3. If the gram atomic weight of aluminum is 26.9815 g, calculate the atom density of the aluminum.

Solution:

$$N = \frac{\rho \ N_A}{M}$$

$$= \frac{2.699 \ \frac{g}{cm^3} \left(6.022 \ x \ 10^{23} \ \frac{atoms}{mole} \right)}{26.9815 \ \frac{g}{mole}}$$

$$= 6.024 \ x \ 10^{22} \ \frac{atoms}{cm^3}$$

Cross Sections

The probability of a neutron interacting with a nucleus for a particular reaction is dependent upon not only the kind of nucleus involved, but also the energy of the neutron. Accordingly, the absorption of a thermal neutron in most materials is much more probable than the absorption of a fast neutron. Also, the probability of interaction will vary depending upon the type of reaction involved.

The probability of a particular reaction occurring between a neutron and a nucleus is called the *microscopic cross section* (σ) of the nucleus for the particular reaction. This cross section will vary with the energy of the neutron. The microscopic cross section may also be regarded as the effective area the nucleus presents to the neutron for the particular reaction. The larger the effective area, the greater the probability for reaction.

Because the microscopic cross section is an area, it is expressed in units of area, or square centimeters. A square centimeter is tremendously large in comparison to the effective area of a nucleus, and it has been suggested that a physicist once referred to the measure of a square centimeter as being "as big as a barn" when applied to nuclear processes. The name has persisted and microscopic cross sections are expressed in terms of *barns*. The relationship between barns and cm^2 is shown below.

1 barn = 10^{-24} cm^2

Whether a neutron will interact with a certain volume of material depends not only on the microscopic cross section of the individual nuclei but also on the number of nuclei within that volume. Therefore, it is necessary to define another kind of cross section known as the macroscopic cross section (Σ). The *macroscopic cross section* is the probability of a given reaction occurring per unit travel of the neutron. Σ is related to the microscopic cross section (σ) by the relationship shown below.

$$\Sigma = N \sigma \qquad (2\text{-}2)$$

where:

Σ = macroscopic cross section (cm^{-1})
N = atom density of material ($atoms/cm^3$)
σ = microscopic cross-section (cm^2)

The difference between the microscopic and macroscopic cross sections is extremely important and is restated for clarity. The microscopic cross section (σ) represents the effective target area that a single nucleus presents to a bombarding particle. The units are given in barns or cm^2. The macroscopic cross section (Σ) represents the effective target area that is presented by all of the nuclei contained in 1 cm^3 of the material. The units are given as 1/cm or cm^{-1}.

A neutron interacts with an atom of the material it enters in two basic ways. It will either interact through a scattering interaction or through an absorption reaction. The probability of a neutron being absorbed by a particular atom is the microscopic cross section for absorption, σ_a. The probability of a neutron scattering off of a particular nucleus is the microscopic cross section for scattering, σ_s. The sum of the microscopic cross section for absorption and the microscopic cross section for scattering is the total microscopic cross section, σ_T.

$$\sigma_T = \sigma_a + \sigma_s$$

Both the absorption and the scattering microscopic cross sections can be further divided. For instance, the scattering cross section is the sum of the elastic scattering cross section (σ_{se}) and the inelastic scattering cross section (σ_{si}).

$$\sigma_s = \sigma_{se} + \sigma_{si}$$

The microscopic absorption cross section (σ_a) includes all reactions except scattering. However, for most purposes it is sufficient to merely separate it into two categories, fission (σ_f) and capture (σ_c). Radiative capture of neutrons was described in the Neutron Interactions chapter of Module 1.

$$\sigma_a = \sigma_f + \sigma_c$$

The variation of absorption cross sections with neutron energy is often complicated. For many elements the absorption cross sections are small, ranging from a fraction of a barn to a few barns for slow (or thermal) neutrons.

For a considerable number of nuclides of moderately high (or high) mass numbers, an examination of the variation of the absorption cross section with the energy of the incident neutron reveals the existence of three regions on a curve of absorption cross section versus neutron energy. This cross section is illustrated in Figure 1. First, the cross section decreases steadily with increasing neutron energy in a low energy region, which includes the thermal range (E < 1 eV). In this region the absorption cross section, which is often high, is inversely proportional to the velocity (v). This region is frequently referred to as the "1/v region," because the absorption cross section is proportional to 1/v, which is the reciprocal of neutron velocity. Following the 1/v region, there occurs the "resonance region" in which the cross sections rise sharply to high values called "resonance peaks" for neutrons of certain energies, and then fall again. These energies are called resonance energies and are a result of the affinity of the nucleus for neutrons whose energies closely match its discrete, quantum energy levels. That is, when the binding energy of a neutron plus the kinetic energy of the neutron are exactly equal to the amount required to raise a compound nucleus from its ground state to a quantum level, resonance absorption occurs. The following example problem further illustrates this point.

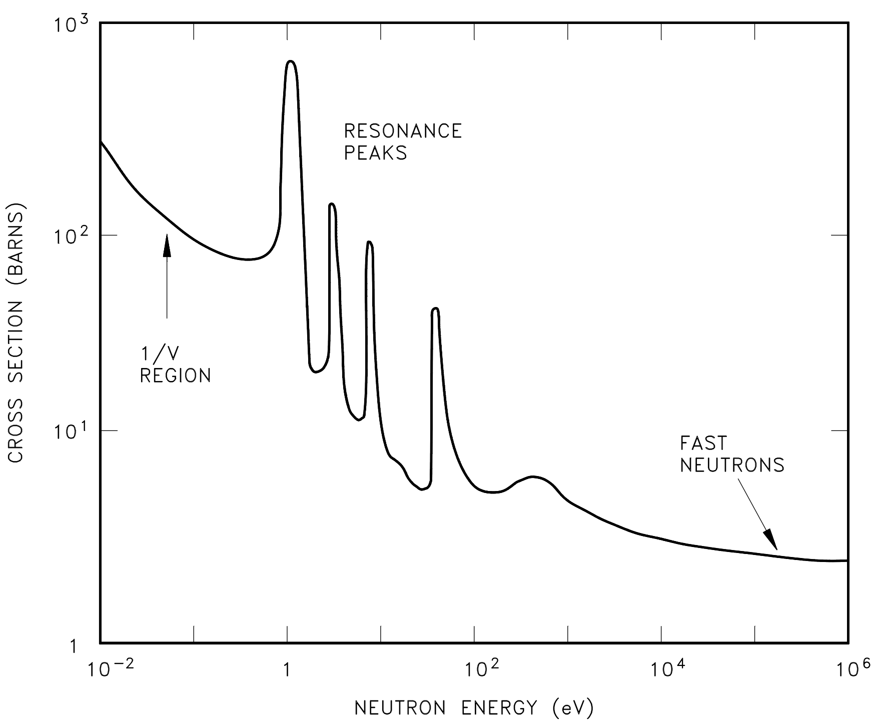

Figure 1 Typical Neutron Absorption Cross Section vs. Neutron Energy

Assuming that uranium-236 has a nuclear quantum energy level at 6.8 MeV above its ground state, calculate the kinetic energy a neutron must possess to undergo resonant absorption in uranium-235 at this resonance energy level.

BE = [Mass(^{235}U) + Mass(neutron) - Mass(^{236}U)] x 931 MeV/amu

BE = (235.043925 + 1.008665 - 236.045563) x 931 MeV/amu

BE = (0.007025 amu) x 931 MeV/amu = 6.54 MeV

6.8 MeV - 6.54 MeV = 0.26 MeV

The difference between the binding energy and the quantum energy level equals the amount of kinetic energy the neutron must possess. The typical heavy nucleus will have many closely-spaced resonances starting in the low energy (eV) range. This is because heavy nuclei are complex and have more possible configurations and corresponding energy states. Light nuclei, being less complex, have fewer possible energy states and fewer resonances that are sparsely distributed at higher energy levels.

For higher neutron energies, the absorption cross section steadily decreases as the energy of the neutron increases. This is called the "fast neutron region." In this region the absorption cross sections are usually less than 10 barns.

With the exception of hydrogen, for which the value is fairly large, the elastic scattering cross sections are generally small, for example, 5 barns to 10 barns. This is close to the magnitude of the actual geometric cross sectional area expected for atomic nuclei. In potential scattering, the cross section is essentially constant and independent of neutron energy. Resonance elastic scattering and inelastic scattering exhibit resonance peaks similar to those associated with absorption cross sections. The resonances occur at lower energies for heavy nuclei than for light nuclei. In general, the variations in scattering cross sections are very small when compared to the variations that occur in absorption cross sections.

Mean Free Path

If a neutron has a certain probability of undergoing a particular interaction in one centimeter of travel, then the inverse of this value describes how far the neutron will travel (in the average case) before undergoing an interaction. This average distance traveled by a neutron before interaction is known as the *mean free path* for that interaction and is represented by the symbol λ. The relationship between the mean free path (λ) and the macroscopic cross section (Σ) is shown below.

$$\lambda = \frac{1}{\Sigma} \tag{2-3}$$

Calculation of Macroscopic Cross Section and Mean Free Path

Most materials are composed of several elements, and because most elements are composed of several isotopes, most materials involve many cross sections, one for each isotope involved. Therefore, to include all the isotopes within a given material, it is necessary to determine the macroscopic cross section for each isotope and then sum all the individual macroscopic cross sections. Equation (2-4) can be used to determine the macroscopic cross section for a composite material.

$$\Sigma = N_1 \sigma_1 + N_2 \sigma_2 + N_3 \sigma_3 + \ldots\ldots\ldots N_n \sigma_n \tag{2-4}$$

where:

N_n = the number of nuclei per cm^3 of the n^{th} element
σ_n = the microscopic cross section of the n^{th} element

The following example problems illustrate the calculation of the macroscopic cross section for a single element and for combinations of materials.

Example 1:

Find the macroscopic thermal neutron absorption cross section for iron, which has a density of 7.86 g/cm^3. The microscopic cross section for absorption of iron is 2.56 barns and the gram atomic weight is 55.847 g.

Solution:

Step 1: Using Equation (2-1), calculate the atom density of iron.

$$N = \frac{\rho\ N_A}{M}$$

$$= \frac{7.86\ \frac{g}{cm^3}\left(6.022\ x\ 10^{23}\ \frac{atoms}{mole}\right)}{55.847\ \frac{g}{mole}}$$

$$= 8.48\ x\ 10^{22}\ \frac{atoms}{cm^3}$$

Step 2: Use this atom density in Equation (2-2) to calculate the macroscopic cross section.

$$\Sigma_a = N\ \sigma_a$$

$$= 8.48\ x\ 10^{22}\ \frac{atoms}{cm^3}\ (2.56\ barns)\left(\frac{1\ x\ 10^{-24}\ cm^2}{1\ barn}\right)$$

$$= 0.217\ cm^{-1}$$

Example 2:

An alloy is composed of 95% aluminum and 5% silicon (by weight). The density of the alloy is 2.66 g/cm^3. Properties of aluminum and silicon are shown below.

Element	Gram Atomic Weight	$\sigma_{a\ (barns)}$	$\sigma_{s\ (barns)}$
Aluminum	26.9815	0.23	1.49
Silicon	28.0855	0.16	2.20

1. Calculate the atom densities for the aluminum and silicon.
2. Determine the absorption and scattering macroscopic cross sections for thermal neutrons.
3. Calculate the mean free paths for absorption and scattering.

Solution:

Step 1: The density of the aluminum will be 95% of the total density. Using Equation (2-1) yields the atom densities.

$$N_{Al} = \frac{\rho_{Al}\ N_A}{M_{Al}}$$

$$= \frac{0.95 \left(2.66\ \frac{g}{cm^3}\right)\left(6.022\ x\ 10^{23}\ \frac{atoms}{mole}\right)}{26.9815\ \frac{g}{mole}}$$

$$= 5.64\ x\ 10^{22}\ \frac{atoms}{cm^3}$$

$$N_{Si} = \frac{\rho_{Si}\ N_A}{M_{Si}}$$

$$= \frac{0.05 \left(2.66\ \frac{g}{cm^3}\right)\left(6.022\ x\ 10^{23}\ \frac{atoms}{mole}\right)}{28.0855\ \frac{g}{mole}}$$

$$= 2.85\ x\ 10^{21}\ \frac{atoms}{cm^3}$$

Step 2: The macroscopic cross sections for absorption and scattering are calculated using Equation (2-4).

$$\Sigma_a = N_{Al}\ \sigma_{a,Al} + N_{Si}\ \sigma_{a,Si}$$

$$= \left(5.64\ x\ 10^{22}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(0.23\ x\ 10^{-24}\ \text{cm}^2\right) + \left(2.85\ x\ 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(0.16\ x\ 10^{-24}\ \text{cm}^2\right)$$

$$= 0.0134\ \text{cm}^{-1}$$

$$\Sigma_s = N_{Al}\ \sigma_{s,Al} + N_{Si}\ \sigma_{s,Si}$$

$$= \left(5.64\ x\ 10^{22}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(1.49\ x\ 10^{-24}\ \text{cm}^2\right) + \left(2.85\ x\ 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(2.20\ x\ 10^{-24}\ \text{cm}^2\right)$$

$$= 0.0903\ \text{cm}^{-1}$$

Step 3: The mean free paths are calculated by inserting the macroscopic cross sections calculated above into Equation (2-3).

$$\lambda_a = \frac{1}{\Sigma_a}$$

$$= \frac{1}{0.01345\ \text{cm}^{-1}}$$

$$= 74.3\ \text{cm}$$

$$\lambda_s = \frac{1}{\Sigma_s}$$

$$= \frac{1}{0.0903\ \text{cm}^{-1}}$$

$$= 11.1\ \text{cm}$$

Thus, a neutron must travel an average of 74.3 cm to interact by absorption in this alloy, but it must travel only 11.1 cm to interact by scattering.

Effects of Temperature on Cross Section

As discussed, the microscopic absorption cross section varies significantly as neutron energy varies. The microscopic cross sections provided on most charts and tables are measured for a standard neutron velocity of 2200 meters/second, which corresponds to an ambient temperature of 68°F. Therefore, if our material is at a higher temperature, the absorption cross section will be lower than the value for 68°F, and any cross sections which involve absorption (for example, σ_a, σ_c, σ_f) must be corrected for the existing temperature.

The following formula is used to correct microscopic cross sections for temperature. Although the example illustrates absorption cross section, the same formula may be used to correct capture and fission cross sections.

$$\sigma = \sigma_o \left(\frac{T_o}{T} \right)^{1/2}$$

where:

σ = microscopic cross section corrected for temperature
σ_o = microscopic cross section at reference temperature (68°F or 20°C)
T_o = reference temperature (68°F) in degrees Rankine (°R) or Kelvin (°K)
T = temperature for which corrected value is being calculated

NOTE: When using this formula, all temperatures must be converted to °R or °K.

$$°R = °F + 460$$
$$°K = °C + 273$$

Example:

What is the value of σ_f for uranium-235 for thermal neutrons at 500°F? Uranium-235 has a σ_f of 583 barns at 68°F.

Solution:

$$\sigma_f = \sigma_{f,o} \left(\frac{T_o}{T} \right)^{1/2}$$

$$= 583 \text{ barns} \left(\frac{68°F + 460}{500°F + 460} \right)^{1/2}$$

$$= 432 \text{ barns}$$

Neutron Flux

Macroscopic cross sections for neutron reactions with materials determine the probability of one neutron undergoing a specific reaction per centimeter of travel through that material. If one wants to determine how many reactions will actually occur, it is necessary to know how many neutrons are traveling through the material and how many centimeters they travel each second.

It is convenient to consider the number of neutrons existing in one cubic centimeter at any one instant and the total distance they travel each second while in that cubic centimeter. The number of neutrons existing in a cm^3 of material at any instant is called *neutron density* and is represented by the symbol n with units of neutrons/cm^3. The total distance these neutrons can travel each second will be determined by their velocity.

A good way of defining *neutron flux* (ϕ) is to consider it to be the total path length covered by all neutrons in one cubic centimeter during one second. Mathematically, this is the equation below.

$$\phi = n\,v \qquad (2\text{-}5)$$

where:

ϕ = neutron flux (neutrons/cm^2-sec)
n = neutron density (neutrons/cm^3)
v = neutron velocity (cm/sec)

The term neutron flux in some applications (for example, cross section measurement) is used as parallel beams of neutrons traveling in a single direction. The *intensity* (I) of a neutron beam is the product of the neutron density times the average neutron velocity. The directional beam intensity is equal to the number of neutrons per unit area and time (neutrons/cm^2-sec) falling on a surface perpendicular to the direction of the beam.

One can think of the neutron flux in a reactor as being comprised of many neutron beams traveling in various directions. Then, the neutron flux becomes the scalar sum of these directional flux intensities (added as numbers and not vectors), that is, $\phi = I_1 + I_2 + I_3 + ... I_n$. Since the atoms in a reactor do not interact preferentially with neutrons from any particular direction, all of these directional beams contribute to the total rate of reaction. In reality, at a given point within a reactor, neutrons will be traveling in all directions.

Self-Shielding

In some locations within the reactor, the flux level may be significantly lower than in other areas due to a phenomenon referred to as *neutron shadowing* or *self-shielding*. For example, the interior of a fuel pin or pellet will "see" a lower average flux level than the outer surfaces since an appreciable fraction of the neutrons will have been absorbed and therefore cannot reach the interior of the fuel pin. This is especially important at resonance energies, where the absorption cross sections are large.

Summary

The important information in this chapter is summarized below.

Nuclear Cross Section and Neutron Flux Summary

- Atom density (N) is the number of atoms of a given type per unit volume of material.
- Microscopic cross section (σ) is the probability of a given reaction occurring between a neutron and a nucleus.
- Microscopic cross sections are measured in units of barns, where 1 barn = 10^{-24} cm^2.
- Macroscopic cross section (Σ) is the probability of a given reaction occurring per unit length of travel of the neutron. The units for macroscopic cross section are cm^{-1}.
- The mean free path (λ) is the average distance that a neutron travels in a material between interactions.
- Neutron flux (ϕ) is the total path length traveled by all neutrons in one cubic centimeter of material during one second.
- The macroscopic cross section for a material can be calculated using the equation below.

  $$\Sigma = N \sigma$$

- The absorption cross section for a material usually has three distinct regions. At low neutron energies (<1 eV) the cross section is inversely proportional to the neutron velocity.
- Resonance absorption occurs when the sum of the kinetic energy of the neutron and its binding energy is equal to an allowed nuclear energy level of the nucleus.
- Resonance peaks exist at intermediate energy levels. For higher neutron energies, the absorption cross section steadily decreases as the neutron energy increases.
- The mean free path equals 1/Σ.
- The macroscopic cross section for a mixture of materials can be calculated using the equation below.

  $$\Sigma = N_1 \sigma_1 + N_2 \sigma_2 + N_3 \sigma_3 + \ldots\ldots\ldots N_n \sigma_n$$

- Self-shielding is where the local neutron flux is depressed within a material due to neutron absorption near the surface of the material.