

SUBCRITICAL MULTIPLICATION

Subcritical multiplication is the phenomenon that accounts for the changes in neutron flux that takes place in a subcritical reactor due to reactivity changes. It is important to understand subcritical multiplication in order to understand reactor response to changes in conditions.

- EO 1.1 DEFINE the following terms:**
- a. Subcritical multiplication**
 - b. Subcritical multiplication factor**
- EO 1.2 Given a neutron source strength and a subcritical system of known k_{eff} , CALCULATE the steady-state neutron level.**
- EO 1.3 Given an initial count rate and k_{eff} , CALCULATE the final count rate that will result from the addition of a known amount of reactivity.**
- EO 1.4 Given count rates vs. the parameter being adjusted, ESTIMATE the value of the parameter at which the reactor will become critical through the use of a 1/M plot.**

Subcritical Multiplication Factor

When a reactor is in a shutdown condition, neutrons are still present to interact with the fuel. These source neutrons are produced by a variety of methods that were discussed in Module 2. If neutrons and fissionable material are present in the reactor, fission will take place. Therefore, a reactor will always be producing a small number of fissions even when it is shutdown.

Consider a reactor in which k_{eff} is 0.6. If 100 neutrons are suddenly introduced into the reactor, these 100 neutrons that start the current generation will produce 60 neutrons (100×0.6) from fission to start the next generation. The 60 neutrons that start the second generation will produce 36 neutrons (60×0.6) to start the third generation. The number of neutrons produced by fission in subsequent generations due to the introduction of 100 source neutrons into the reactor is shown below.

Generation	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
Neutrons	100	60	36	22	13	8	5	3	2	1	0	0

Because the reactor is subcritical, neutrons introduced in the reactor will have a decreasing effect on each subsequent generation. The addition of source neutrons to the reactor containing fissionable material has the effect of maintaining a much higher stable neutron level due to the fissions occurring than the neutron level that would result from the source neutrons alone. The effects of adding source neutrons at a rate of 100 neutrons per generation to a reactor with a k_{eff} of 0.6 are shown below.

Generation	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
	100	60	36	22	13	8	5	3	2	1	0	0
		100	60	36	22	13	8	5	3	2	1	0
			100	60	36	22	13	8	5	3	2	1
				100	60	36	22	13	8	5	3	2
					100	60	36	22	13	8	5	3
						100	60	36	22	13	8	5
							100	60	36	22	13	8
								100	60	36	22	13
									100	60	36	22
										100	60	36
											100	60
												100
Total n	100	160	196	218	231	239	244	247	249	250	250	...

A neutron source strength of 100 neutrons per generation will result in 250 neutrons per generation being produced from a combination of sources and fission in a shutdown reactor with a k_{eff} of 0.6. If the value of k_{eff} were higher, the source neutrons would produce a greater number of fission neutrons and their effects would be felt for a larger number of subsequent generations after their addition to the reactor.

The effect of fissions in the fuel increasing the effective source strength of a reactor with a k_{eff} of less than one is *subcritical multiplication*. For a given value of k_{eff} there exists a *subcritical multiplication factor* (M) that relates the source level to the steady-state neutron level of the core. If the value of k_{eff} is known, the amount that the neutron source strength will be multiplied (M) can easily be determined by Equation (4-1).

$$M = \frac{1}{1 - k_{\text{eff}}} \quad (4-1)$$

Example:

Calculate the subcritical multiplication factors for the following values of k_{eff} .

- 1) $k_{\text{eff}} = 0.6$
- 2) $k_{\text{eff}} = 0.986$

Solution:

1)

$$\begin{aligned} M &= \frac{1}{1 - k_{\text{eff}}} \\ &= \frac{1}{1 - 0.6} \\ &= 2.5 \end{aligned}$$

2)

$$\begin{aligned} M &= \frac{1}{1 - k_{\text{eff}}} \\ &= \frac{1}{1 - 0.986} \\ &= 71.4 \end{aligned}$$

The example above illustrates that the subcritical multiplication factor will increase as positive reactivity is added to a shutdown reactor, increasing the value of k_{eff} . If the source strength of this reactor were 1000 neutrons/sec, the neutron level would increase from 2500 neutrons/second at a k_{eff} of 0.6 to a neutron level of 71,400 neutrons/sec at a k_{eff} of 0.986.

Effect of Reactivity Changes on Subcritical Multiplication

In a subcritical reactor, the neutron level is related to the source strength by Equation (4-2).

$$N = (S) (M) \tag{4-2}$$

where:

N	=	neutron level
S	=	neutron source strength
M	=	subcritical multiplication factor

If the term M in Equation (4-2) is replaced by the expression $1/1-k_{\text{eff}}$ from Equation (4-1), the following expression results.

$$N = S \left(\frac{1}{1 - k_{\text{eff}}} \right) \quad (4-3)$$

Example:

A reactor contains a neutron source that produces 110,000 neutrons per second. The reactor has a k_{eff} of 0.986. Calculate the stable total neutron production rate in the reactor.

Solution:

The neutron production rate is calculated using Equation (4-3).

$$\begin{aligned} N &= S \left(\frac{1}{1 - k_{\text{eff}}} \right) \\ &= 110,000 \frac{\text{neutrons}}{\text{second}} \left(\frac{1}{1 - 0.986} \right) \\ &= 7.86 \times 10^6 \frac{\text{neutrons}}{\text{second}} \end{aligned}$$

To this point it has been necessary to know the neutron source strength of the reactor in order to use the concept of subcritical multiplication. In most reactors the actual strength of the neutron sources is difficult, if not impossible, to determine. Even though the actual source strength may not be known, it is still possible to relate the change in reactivity to a change in neutron level.

Consider a reactor at two different times when k_{eff} is two different values, k_1 and k_2 . The neutron level at each time can be determined based on the neutron source strength and the subcritical multiplication factor using Equation (4-3).

$$N_1 = S \left(\frac{1}{1 - k_1} \right) \quad N_2 = S \left(\frac{1}{1 - k_2} \right)$$

The equation for N_1 can be divided by the equation for N_2 .

$$\frac{N_1}{N_2} = \frac{S \left(\frac{1}{1 - k_1} \right)}{S \left(\frac{1}{1 - k_2} \right)}$$
$$\frac{N_1}{N_2} = \frac{1 - k_2}{1 - k_1}$$

Because the source strength appears in both the numerator and denominator, it cancels out of the equation. Therefore, the neutron level at any time can be determined based on the neutron level present at any other time provided the values of k_{eff} or reactivity for both times are known.

The neutron level in a shutdown reactor is typically monitored using instruments that measure the neutron leakage out of the reactor. The neutron leakage is proportional to the neutron level in the reactor. Typical units for displaying the instrument reading are counts per second (cps). Because the instrument count rate is proportional to the neutron level, the above equation can be restated as shown in Equation (4-4).

$$\frac{CR_1}{CR_2} = \frac{1 - k_2}{1 - k_1} \quad (4-4)$$

where:

$$\begin{aligned} CR_1 &= \text{count rate at time 1} \\ CR_2 &= \text{count rate at time 2} \\ k_1 &= k_{\text{eff}} \text{ at time 1} \\ k_2 &= k_{\text{eff}} \text{ at time 2} \end{aligned}$$

Equation (4-4) is very useful during the shutdown operation of a reactor. Before adding positive reactivity to a reactor, it is possible to predict the effect the reactivity addition will have on the neutron level.

Example:

A reactor that has a reactivity of -1000 pcm has a count rate of 42 counts per second (cps) on the neutron monitoring instrumentation. Calculate what the neutron level should be after a positive reactivity insertion of 500 pcm from the withdrawal of control rods.

Solution:

Step 1: Determine the initial value of k_{eff} for the core.

$$\begin{aligned} k_1 &= \frac{1}{1 - \rho_1} \\ &= \frac{1}{1 - (-0.01000)} \\ &= 0.9901 \end{aligned}$$

Step 2: Determine the final value of k_{eff} for the core. The final value of reactivity will be -500 pcm (-1000 + 500).

$$\begin{aligned} k_2 &= \frac{1}{1 - \rho_2} \\ &= \frac{1}{1 - (-0.00500)} \\ &= 0.9950 \end{aligned}$$

Step 3: Use Equation (4-4) to determine the final count rate.

$$\begin{aligned} \frac{CR_1}{CR_2} &= \frac{1 - k_2}{1 - k_1} \\ CR_2 &= CR_1 \left(\frac{1 - k_1}{1 - k_2} \right) \\ &= 42 \text{ cps} \left(\frac{1 - 0.9901}{1 - 0.9950} \right) \\ &= 83 \text{ cps} \end{aligned}$$

Notice from this example that the count rate doubled as the reactivity was halved (e.g., reactivity was changed from -1000 pcm to -500 pcm).

Use of 1/M Plots

Because the subcritical multiplication factor is related to the value of k_{eff} , it is possible to monitor the approach to criticality through the use of the subcritical multiplication factor. As positive reactivity is added to a subcritical reactor, k_{eff} will get nearer to one. As k_{eff} gets nearer to one, the subcritical multiplication factor (M) gets larger. The closer the reactor is to criticality, the faster M will increase for equal step insertions of positive reactivity. When the reactor becomes critical, M will be infinitely large. For this reason, monitoring and plotting M during an approach to criticality is impractical because there is no value of M at which the reactor clearly becomes critical.

Instead of plotting M directly, its inverse ($1/M$) is plotted on a graph of $1/M$ versus rod height.

$$M = \frac{1}{1 - k_{\text{eff}}}$$

$$\frac{1}{M} = 1 - k_{\text{eff}}$$

As control rods are withdrawn and k_{eff} approaches one and M approaches infinity, $1/M$ approaches zero. For a critical reactor, $1/M$ is equal to zero. A true $1/M$ plot requires knowledge of the neutron source strength. Because the actual source strength is usually unknown, a reference count rate is substituted, and the calculation of the factor $1/M$ is through the use of Equation (4-5).

$$\frac{1}{M} = \frac{\text{CR}_o}{\text{CR}} \quad (4-5)$$

where:

$1/M$	=	inverse multiplication factor
CR_o	=	reference count rate
CR	=	current count rate

In practice, the reference count rate used is the count rate prior to the beginning of the reactivity change. The startup procedures for many reactors include instructions to insert positive reactivity in incremental steps with delays between the reactivity insertions to allow time for subcritical multiplication to increase the steady-state neutron population to a new, higher level and allow more accurate plotting of $1/M$. The neutron population will typically reach its new steady-state value within 1-2 minutes, but the closer the reactor is to criticality, the longer the time will be to stabilize the neutron population.

Example:

Given the following rod withdrawal data, construct a $1/M$ plot and estimate the rod position when criticality would occur. The initial count rate on the nuclear instrumentation prior to rod withdrawal is 50 cps.

Rod Withdrawal (inches)	Count Rate (cps)
2	55
4	67
6	86
8	120
10	192
12	500

Solution:

Step 1: Calculate $1/M$ for each of the rod positions using equation (4-5). The reference count rate is 50 cps at a rod position of zero.

Rod Withdrawal (inches)	Count Rate (cps)	CR_0/CR
0	50	1
2	55	0.909
4	67	0.746
6	86	0.581
8	120	0.417
10	192	0.260
12	500	0.100

Step 2: Plotting these values, as shown in Figure 1, and extrapolating to a $1/M$ value of 0 reveals that the reactor will go critical at approximately 13 inches of rod withdrawal.

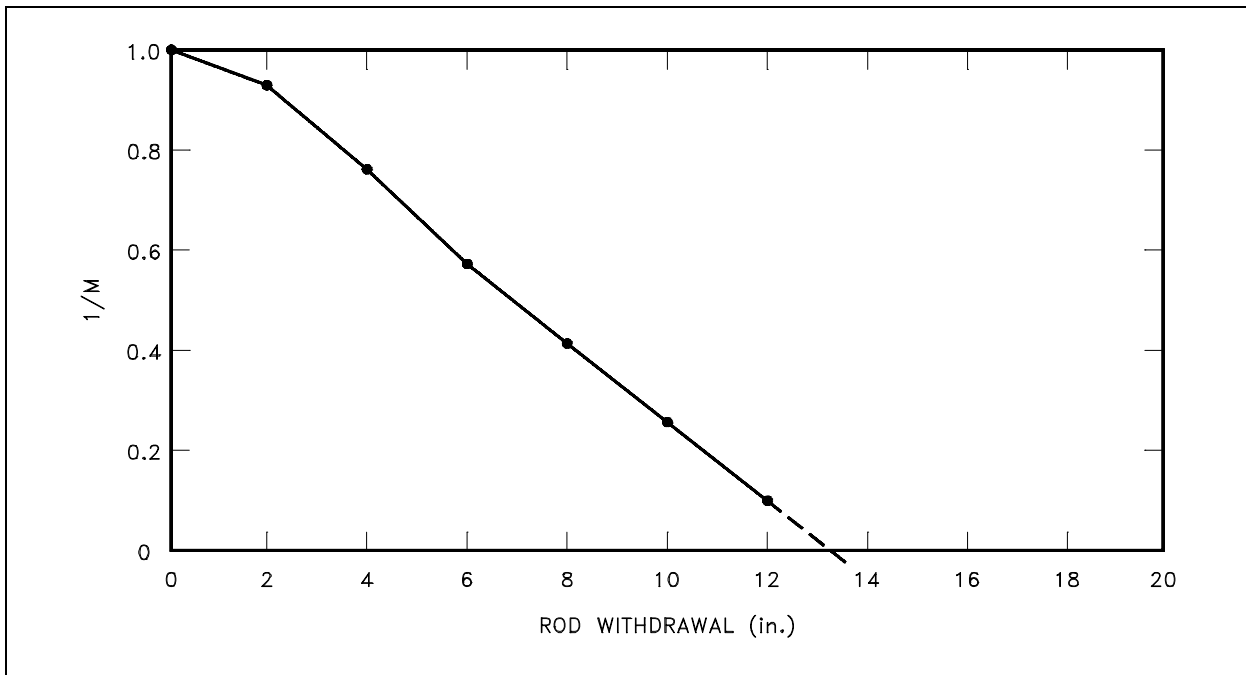


Figure 1 $1/M$ Plot vs. Rod Withdrawal

Summary

The important information in this chapter is summarized below.

Subcritical Multiplication Summary

- Subcritical multiplication is the effect of fissions in the fuel increasing the effective source strength of a reactor with a k_{eff} less than one.
- Subcritical multiplication factor is the factor that relates the source level to the steady-state neutron level of the core.
- The steady-state neutron level of a subcritical reactor can be calculated based on the source strength and k_{eff} using Equation (4-3).

$$N = S \left(\frac{1}{1 - k_{\text{eff}}} \right)$$

- The count rate expected in a subcritical reactor following a change in reactivity can be calculated based on the initial count rate, initial k_{eff} , and amount of reactivity addition using Equation (4-4).

$$\frac{CR_1}{CR_2} = \frac{1 - k_2}{1 - k_1}$$

- 1/M plots can be used to predict the point of criticality.