REACTOR KINETICS

The response of neutron flux and reactor power to changes in reactivity is much different in a critical reactor than in a subcritical reactor. The reliance of the chain reaction on delayed neutrons makes the rate of change of reactor power controllable.

EO 2.1 DEFINE the following terms:

- **a. Reactor period**
- **b. Doubling time**
- **c. Reactor startup rate**

EO 2.2 DESCRIBE the relationship between the delayed neutron fraction, average delayed neutron fraction, and effective delayed neutron fraction.

EO 2.3 WRITE the period equation and IDENTIFY each symbol.

EO 2.4 Given the reactivity of the core and values for the effective average delayed neutron fraction and decay constant, CALCULATE the reactor period and the startup rate.

EO 2.5 Given the initial power level and either the doubling or halving time, CALCULATE the power at any later time.

EO 2.6 Given the initial power level and the reactor period, CALCULATE the power at any later time.

EO 2.7 EXPLAIN what is meant by the terms prompt drop and prompt jump.

EO 2.8 DEFINE the term prompt critical.

EO 2.9 DESCRIBE reactor behavior during the prompt critical condition.

EO 2.10 EXPLAIN the use of measuring reactivity in units of dollars.

Reactor Period (τ)

The *reactor period* is defined as the time required for reactor power to change by a factor of "e," where "e" is the base of the natural logarithm and is equal to about 2.718. The reactor period is usually expressed in units of seconds. From the definition of reactor period, it is possible to develop the relationship between reactor power and reactor period that is expressed by Equation (4-6).

$$P = P_o e^{t/\tau} \tag{4-6}$$

where:

- P = transient reactor power
- P_o = initial reactor power
- τ = reactor period (seconds)
- t = time during the reactor transient (seconds)

The smaller the value of τ, the more rapid the change in reactor power. If the reactor period is positive, reactor power is increasing. If the reactor period is negative, reactor power is decreasing.

There are numerous equations used to express reactor period, but Equation (4-7) shown below, or portions of it, will be useful in most situations. The first term in Equation (4-7) is the prompt term and the second term is the delayed term.

$$\tau = \frac{\ell^*}{\rho} + \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho + \dot{\rho}} \tag{4-7}$$

where:

- ℓ^* = prompt generation lifetime
- $\bar{\beta}_{eff}$ = effective delayed neutron fraction
- ρ = reactivity
- λ_{eff} = effective delayed neutron precursor decay constant
- $\dot{\rho}$ = rate of change of reactivity

Effective Delayed Neutron Fraction

Recall that β, the *delayed neutron fraction*, is the fraction of all fission neutrons that are born as delayed neutrons. The value of β depends upon the actual nuclear fuel used. As discussed in Module 1, the delayed neutron precursors for a given type of fuel are grouped on the basis of half-life. The following table lists the fractional neutron yields for each delayed neutron group of three common types of fuel.

TABLE 1
Delayed Neutron Fractions for Various Fuels

Group	Half-Life (sec)	Uranium-235	Uranium-238	Plutonium-239
1	55.6	0.00021	0.0002	0.00021
2	22.7	0.00141	0.0022	0.00182
3	6.22	0.00127	0.0025	0.00129
4	2.30	0.00255	0.0061	0.00199
5	0.61	0.00074	0.0035	0.00052
6	0.23	0.00027	0.0012	0.00027
TOTAL	-	0.00650	0.0157	0.00200

The term $\bar{\beta}$ (pronounced beta-bar) is the *average delayed neutron fraction*. The value of $\bar{\beta}$ is the weighted average of the total delayed neutron fractions of the individual types of fuel. Each total delayed neutron fraction value for each type of fuel is weighted by the percent of total neutrons that the fuel contributes through fission. If the percentage of fissions occurring in the different types of fuel in a reactor changes over the life of the core, the average delayed neutron fraction will also change. For a light water reactor using low enriched fuel, the average delayed neutron fraction can change from 0.0070 to 0.0055 as uranium-235 is burned out and plutonium-239 is produced from uranium-238.

Delayed neutrons do not have the same properties as prompt neutrons released directly from fission. The average energy of prompt neutrons is about 2 MeV. This is much greater than the average energy of delayed neutrons (about 0.5 MeV). The fact that delayed neutrons are born at lower energies has two significant impacts on the way they proceed through the neutron life cycle. First, delayed neutrons have a much lower probability of causing fast fissions than prompt neutrons because their average energy is less than the minimum required for fast fission to occur. Second, delayed neutrons have a lower probability of leaking out of the core while they are at fast energies, because they are born at lower energies and subsequently travel a shorter distance as fast neutrons. These two considerations (lower fast fission factor and higher fast non-leakage probability for delayed neutrons) are taken into account by a term called the *importance factor* (I). The importance factor relates the average delayed neutron fraction to the effective delayed neutron fraction.

The *effective delayed neutron fraction* $\left(\bar{\beta}_{eff}\right)$ is defined as the fraction of neutrons at thermal energies which were born delayed. The effective delayed neutron fraction is the product of the average delayed neutron fraction and the importance factor.

$$\bar{\beta}_{eff} = \bar{\beta}\ I$$

where:

$\bar{\beta}_{eff}$ = effective delayed neutron fraction

$\bar{\beta}$ = average delayed neutron fraction

I = importance factor

In a small reactor with highly enriched fuel, the increase in fast non-leakage probability will dominate the decrease in the fast fission factor, and the importance factor will be greater than one. In a large reactor with low enriched fuel, the decrease in the fast fission factor will dominate the increase in the fast non-leakage probability and the importance factor will be less than one (about 0.97 for a commercial PWR).

Effective Delayed Neutron Precursor Decay Constant

Another new term has been introduced in the reactor period (τ) equation. That term is λ_{eff} (pronounced lambda effective), the *effective delayed neutron precursor decay constant*. The decay rate for a given delayed neutron precursor can be expressed as the product of precursor concentration and the decay constant (λ) of that precursor. The decay constant of a precursor is simply the fraction of an initial number of the precursor atoms that decays in a given unit time. A decay constant of 0.1 sec^{-1}, for example, implies that one-tenth, or ten percent, of a sample of precursor atoms decays within one second. The value for the effective delayed neutron precursor decay constant, λ_{eff}, varies depending upon the balance existing between the concentrations of the precursor groups and the nuclide(s) being used as the fuel.

If the reactor is operating at a constant power, all the precursor groups reach an equilibrium value. During an up-power transient, however, the shorter-lived precursors decaying at any given instant were born at a higher power level (or flux level) than the longer-lived precursors decaying at the same instant. There is, therefore, proportionately more of the shorter-lived and fewer of the longer-lived precursors decaying at that given instant than there are at constant power. The value of λ_{eff} is closer to that of the shorter-lived precursors.

During a down-power transient the longer-lived precursors become more significant. The longer-lived precursors decaying at a given instant were born at a higher power level (or flux level) than the shorter-lived precursors decaying at that instant. Therefore, proportionately more of the longer-lived precursors are decaying at that instant, and the value of λ_{eff} approaches the values of the longer-lived precursors.

Approximate values for λ_{eff} are 0.08 sec^{-1} for steady-state operation, 0.1 sec^{-1} for a power increase, and 0.05 sec^{-1} for a power decrease. The exact values will depend upon the materials used for fuel and the value of the reactivity of the reactor core.

Returning now to Equation (4-7) for reactor period.

$$\tau \; = \; \underbrace{\frac{\ell^*}{\rho}}_{\begin{pmatrix}\text{prompt}\\ \text{term}\end{pmatrix}} \; + \; \underbrace{\frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\, \rho \; + \; \dot{\rho}}}_{\begin{pmatrix}\text{delayed}\\ \text{term}\end{pmatrix}}$$

If the positive reactivity added is less than the value of $\bar{\beta}_{eff}$, the emission of prompt fission neutrons alone is not sufficient to overcome losses to non-fission absorption and leakage. If delayed neutrons were not being produced, the neutron population would decrease as long as the reactivity of the core has a value less than the effective delayed neutron fraction. The positive reactivity insertion is followed immediately by a small immediate power increase called the *prompt jump*. This power increase occurs because the rate of production of prompt neutrons changes abruptly as the reactivity is added. Recall from an earlier module that the generation time for prompt neutrons is on the order of 10^{-13} seconds. The effect can be seen in Figure 2. After the prompt jump, the rate of change of power cannot increase any more rapidly than the built-in time delay the precursor half-lives allow. Therefore, the power rise is controllable, and the reactor can be operated safely.

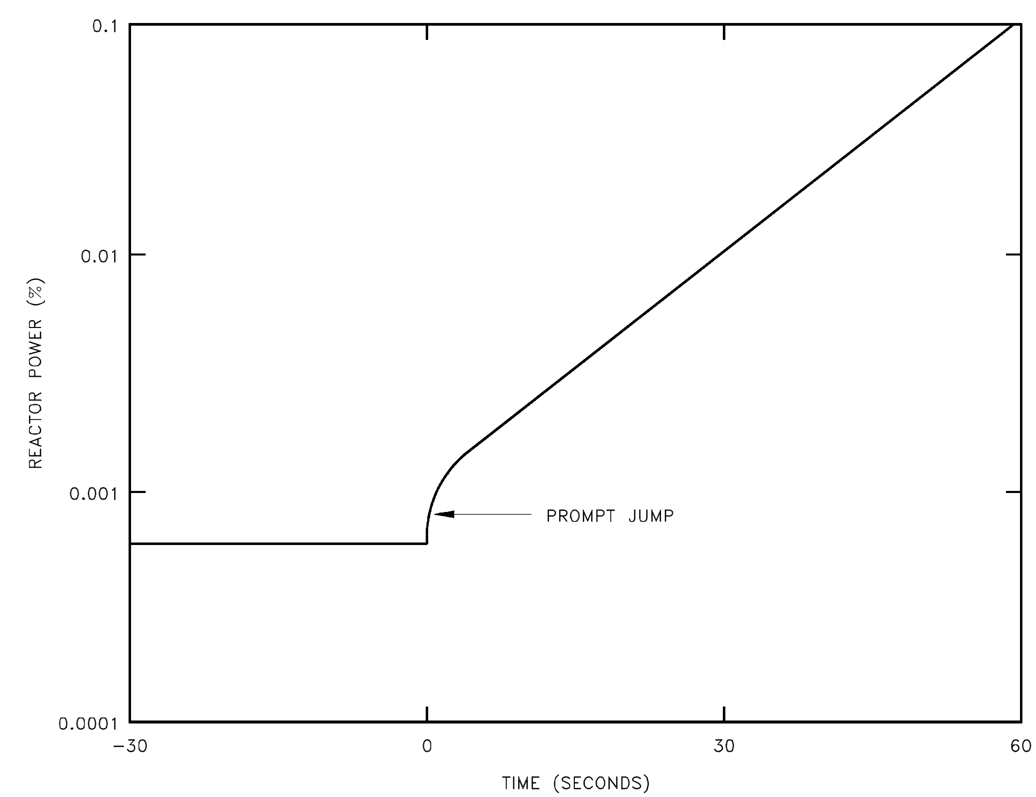

Figure 2 Reactor Power Response to Positive Reactivity Addition

Conversely, in the case where negative reactivity is added to the core there will be a prompt drop in reactor power. The *prompt drop* is the small immediate decrease in reactor power caused by the negative reactivity addition. The prompt drop is illustrated in Figure 3. After the prompt drop, the rate of change of power slows and approaches the rate determined by the delayed term of Equation (4-7).

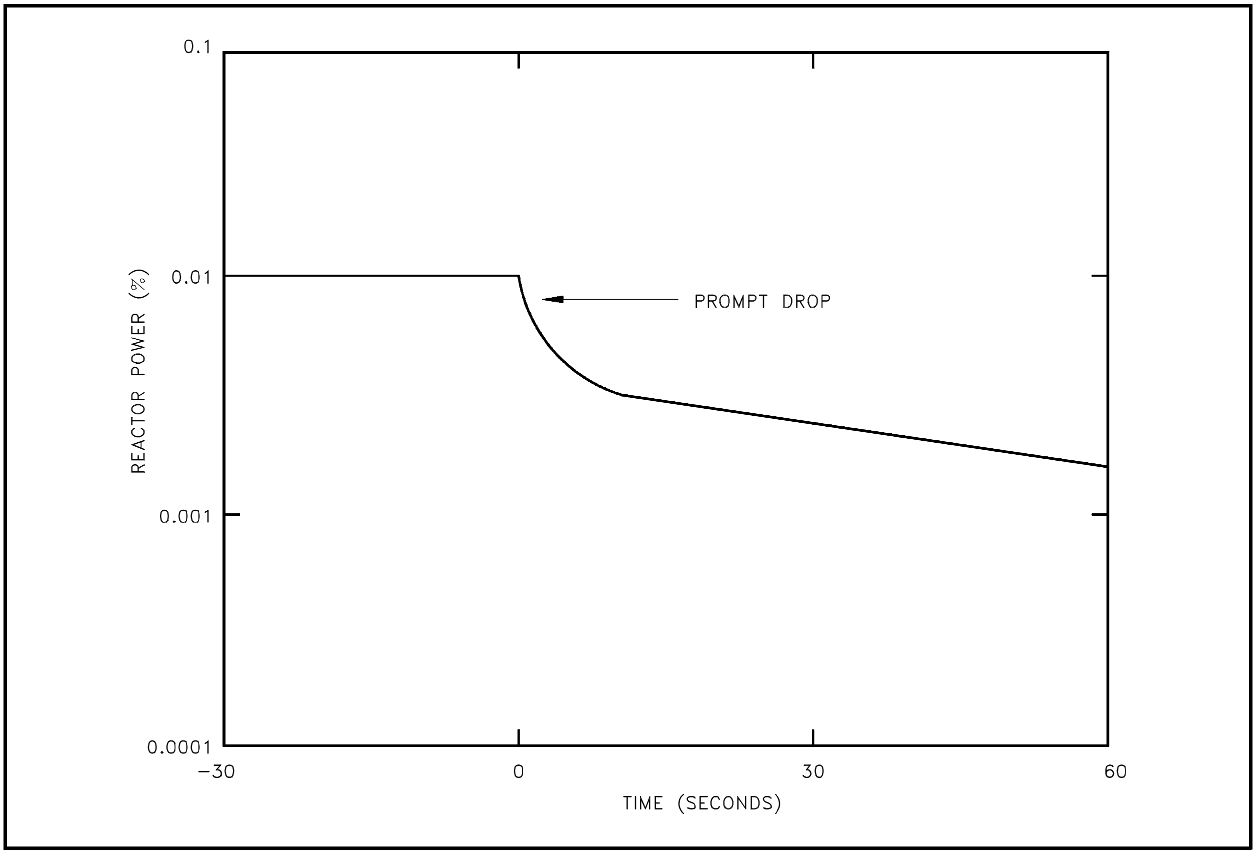

Figure 3 Reactor Power Response to Negative Reactivity Addition

Prompt Criticality

It can be readily seen from Equation (4-7) that if the amount of positive reactivity added equals the value of $\bar{\beta}_{eff}$, the reactor period equation becomes the following.

$$\tau = \frac{\ell^*}{\rho}$$

In this case, the production of prompt neutrons alone is enough to balance neutron losses and increase the neutron population. The condition where the reactor is critical on prompt neutrons, and the neutron population increases as rapidly as the prompt neutron generation lifetime allows is known as *prompt critical*. The prompt critical condition does not signal a dramatic change in neutron behavior. The reactor period changes in a regular manner between reactivities above and below this reference. Prompt critical is, however, a convenient condition for marking the transition from delayed neutron to prompt neutron time scales. A reactor whose reactivity even approaches prompt critical is likely to suffer damage due to the rapid rise in power to a very high level. For example, a reactor which has gone prompt critical could experience a several thousand percent power increase in less than one second.

Because the prompt critical condition is so important, a specific unit of reactivity has been defined that relates to it. The unit of reactivity is the dollar ($), where one dollar of reactivity is equivalent to the effective delayed neutron fraction $\left(\bar{\beta}_{eff}\right)$. A reactivity unit related to the dollar is the cent, where one cent is one-hundredth of a dollar. If the reactivity of the core is one dollar, the reactor is prompt critical. Because the effective delayed neutron fraction is dependent upon the nuclides used as fuel, the value of the dollar is also dependent on the nuclides used as fuel.

Stable Period Equation

For normal reactor operating conditions, the value of positive reactivity in the reactor is never permitted to approach the effective delayed neutron fraction, and the reactor period equation is normally written as follows.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho + \dot{\rho}} \tag{4-8}$$

Equation (4-8) is referred to as the *transient period equation* since it incorporates the $\dot{\rho}$ term to account for the changing amount of reactivity in the core. The ℓ^*/ρ term (prompt period) is normally negligible with respect to the remainder of the equation and is often not included.

For conditions when the amount of reactivity in the core is constant ($\dot{\rho} = 0$), and the reactor period is unchanging, Equation (4-8) can be simplified further to Equation (4-9) which is known as the stable period equation.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho} \tag{4-9}$$

Reactor Startup Rate (SUR)

The *reactor startup rate* (SUR) is defined as the number of factors of ten that power changes in one minute. The units of SUR are powers of ten per minute, or decades per minute (DPM). Equation (4-10) shows the relationship between reactor power and startup rate.

$$P = P_o \, 10^{SUR\,(t)} \tag{4-10}$$

where:

SUR = reactor startup rate (DPM)
t = time during reactor transient (minutes)

The relationship between reactor period and startup rate can be developed by considering Equations (4-6) and (4-10).

$$P = P_o \, e^{t/\tau} \quad \text{and} \quad P = P_o \, 10^{SUR\,(t)}$$

$$\frac{P}{P_o} = e^{t/\tau} = 10^{SUR\,(t)}$$

Changing the base of the exponential term on the right side to "e" ($10 = e^{2.303}$) and solving the result yields the following.

$$e^{t\,(sec)/\tau} = e^{2.303\;SUR\;(t\;(min))}$$

$$\frac{t\;(sec)}{\tau} = 2.303\;SUR(t\;(min))$$

$$\frac{60}{\tau} = 2.303\;SUR$$

$$SUR = \frac{26.06}{\tau} \tag{4-11}$$

Doubling Time

Sometimes it is useful to discuss the rate of change of reactor power in terms similar to those used in radioactive decay calculations. *Doubling or halving time* are terms that relate to the amount of time it takes reactor power to double or be reduced to one-half the initial power level. If the stable reactor period is known, doubling time can be determined as follows.

Doubling time (DT) = τ (ln 2)

where:

τ = stable reactor period
ln 2 = natural logarithm of 2

When the doubling time is known, the power level change from P_o is given by the following equation.

$$P = P_o \, 2^{t/DT} \tag{4-12}$$

where:

t = time interval of transient
DT = doubling time

The following example problems reinforce the concepts of period and startup rate.

Example 1:

A reactor has a λ_{eff} of 0.10 sec^{-1} and an effective delayed neutron fraction of 0.0070. If k_{eff} is equal to 1.0025, what is the stable reactor period and the SUR?

Solution:

Step 1: First solve for reactivity using Equation (3-5).

$$\rho = \frac{k_{eff} - 1}{k_{eff}}$$

$$= \frac{1.0025 - 1}{1.0025}$$

$$= 0.00249 \ \Delta k/k$$

Step 2: Use this value of reactivity in Equation (4-9) to calculate reactor period.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff} \, \rho}$$

$$= \frac{0.0070 - 0.00249}{\left(0.10 \ sec^{-1}\right) 0.00249}$$

$$= 18.1 \ sec$$

Step 3: The startup rate can be calculated from the reactor period using Equation (4-11).

$$\begin{aligned} SUR &= \frac{26.06}{\tau} \\ &= \frac{26.06}{18.1 \ \text{sec}} \\ &= 1.44 \ \text{DPM} \end{aligned}$$

Example 2:

130 pcm of negative reactivity is added to a reactor that is initially critical at a power of 100 watts. λ_{eff} for the reactor is 0.05 sec^{-1} and the effective delayed neutron fraction is 0.0068. Calculate the steady state period and startup rate. Also calculate the power level 2 minutes after the reactivity insertion.

Solution:

Step 1: Use Equation (4-9) to calculate the reactor period.

$$\begin{aligned} \tau &= \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff} \ \rho} \\ &= \frac{0.0068 - (-0.00130)}{\left(0.05 \ \text{sec}^{-1}\right) (-0.00130)} \\ &= -124.6 \ \text{sec} \end{aligned}$$

Step 2: The startup rate can be calculated from the reactor period using Equation (4-11).

$$\begin{aligned} SUR &= \frac{26.06}{\tau} \\ &= \frac{26.06}{-124.6 \ \text{sec}} \\ &= -0.2091 \ \text{DPM} \end{aligned}$$

Step 3: Use either Equation (4-1) or Equation (4-10) to calculate the reactor power two minutes after the reactivity insertion.

$$\begin{aligned} P &= P_o \ e^{t/\tau} \\ &= \left(100 \ \text{W}\right) \ e^{(120 \ \text{s}/-124.6 \ \text{s})} \\ &= 38.2 \ \text{W} \end{aligned}$$

$$\begin{aligned} P &= P_o \ 10^{SUR \ (t)} \\ &= (100 \ \text{W}) \ 10^{(-0.2091 \ \text{DPM}) \ (2 \ \text{min})} \\ &= 38.2 \ \text{W} \end{aligned}$$

Example 3:

A reactor has a power level of 1000 watts and a doubling time of 2 minutes. What is the reactor power level 10 minutes later?

Solution:

Use Equation (4-12) to calculate the final power level.

$$P = P_o \ (2)^{t/DT}$$

$$= (1{,}000 \ W) \ (2)^{10 \ min/2 \ min}$$

$$= 32{,}000 \ W$$

Summary

The important information in this chapter is summarized below.

Reactor Kinetics Summary

- Reactor period is the time required for reactor power to change by a factor of e (2.718).
- Doubling time is the time required for reactor power to double.
- Reactor startup rate is the number of factors of ten that reactor power changes in one minute.
- The delayed neutron fraction (β) is the fraction of all fission neutrons that are born as delayed neutrons for a particular type of fuel (that is, uranium-235 and plutonium-239).
- The average delayed neutron fraction ($\bar{\beta}$) is the weighted average of the total delayed neutron fractions of the different types of fuel used in a particular reactor.
- The effective delayed neutron fraction ($\bar{\beta}_{eff}$) is the average delayed neutron fraction multiplied by an Importance Factor which accounts for the fact that delayed neutrons are born at lower average energies than fast neutrons.
- The reactor period equation is stated below.

$$\tau = \underbrace{\frac{\ell^*}{\rho}}_{\text{(prompt term)}} + \underbrace{\frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho + \dot{\rho}}}_{\text{(delayed term)}}$$

where:

τ = reactor period
ℓ^* = prompt generation lifetime
$\bar{\beta}_{eff}$ = effective delayed neutron fraction
ρ = reactivity
λ_{eff} = effective delayed neutron precursor decay constant
$\dot{\rho}$ = rate of change of reactivity

Reactor Kinetics Summary (Cont.)

- Equations (4-9) and (4-11) can be used to calculate the stable reactor period and startup rate.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\ \rho} \qquad SUR = \frac{26.06}{\tau}$$

- The concept of doubling time can be used in a similar manner to reactor period to calculate changes in reactor power using Equation (4-12).

$$P = P_o\ 2^{t/DT}$$

- The reactor period or the startup rate can be used to determine the reactor power using Equations (4-6) and (4-10).

$$P = P_o\ e^{t/\tau} \qquad P = P_o\ 10^{SUR\ (t)}$$

- Prompt jump is the small, immediate power increase that follows a positive reactivity insertion related to an increase in the prompt neutron population.
- Prompt drop is the small, immediate power decrease that follows a negative reactivity insertion related to a decrease in the prompt neutron population.
- Prompt critical is the condition when the reactor is critical on prompt neutrons alone.
- When a reactor is prompt critical, the neutron population, and hence power, can increase as quickly as the prompt neutron generation time allows.
- Measuring reactivity in units of dollars is useful when determining if a reactor is prompt critical. A reactor that contains one dollar of positive reactivity is prompt critical since one dollar of reactivity is equivalent to λ_{eff}.