NEUTRON LIFE CYCLE

Some number of the fast neutrons produced by fission in one generation will eventually cause fission in the next generation. The series of steps that fission neutrons go through as they slow to thermal energies and are absorbed in the reactor is referred to as the neutron life cycle. The neutron life cycle is markedly different between fast reactors and thermal reactors. This chapter presents the neutron life cycle for thermal reactors.

EO 1.1 DEFINE the following terms:

a. Infinite multiplication factor, \( k_\infty \)

b. Effective multiplication factor, \( k_{\text{eff}} \)

c. Subcritical

d. Critical

e. Supercritical

EO 1.2 DEFINE each term in the six factor formula using the ratio of the number of neutrons present at different points in the neutron life cycle.

EO 1.3 Given the macroscopic cross sections for various materials, CALCULATE the thermal utilization factor.

EO 1.4 Given microscopic cross sections for absorption and fission, atom density, and \( \nu \), CALCULATE the reproduction factor.

EO 1.5 Given the numbers of neutrons present at the start of a generation and values for each factor in the six factor formula, CALCULATE the number of neutrons that will be present at any point in the life cycle.

EO 1.6 LIST physical changes in the reactor core that will have an effect on the thermal utilization factor, reproduction factor, or resonance escape probability.

EO 1.7 EXPLAIN the effect that temperature changes will have on the following factors:

a. Thermal utilization factor

b. Resonance escape probability

c. Fast non-leakage probability

d. Thermal non-leakage probability
Infinite Multiplication Factor, $k_\infty$

Not all of the neutrons produced by fission will have the opportunity to cause new fissions because some neutrons will be absorbed by non-fissionable material. Some will be absorbed parasitically in fissionable material and will not cause fission, and others will leak out of the reactor. For the maintenance of a self-sustaining chain reaction, however, it is not necessary that every neutron produced in fission initiate another fission. The minimum condition is for each nucleus undergoing fission to produce, on the average, at least one neutron that causes fission of another nucleus. This condition is conveniently expressed in terms of a multiplication factor.

The number of neutrons absorbed or leaking out of the reactor will determine the value of this multiplication factor, and will also determine whether a new generation of neutrons is larger, smaller, or the same size as the preceding generation. Any reactor of a finite size will have neutrons leak out of it. Generally, the larger the reactor, the lower the fraction of neutron leakage. For simplicity, we will first consider a reactor that is infinitely large, and therefore has no neutron leakage. A measure of the increase or decrease in neutron flux in an infinite reactor is the infinite multiplication factor, $k_\infty$. The infinite multiplication factor is the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption in the preceding generation. This can be expressed mathematically as shown below.

$$k_\infty = \frac{\text{neutron production from fission in one generation}}{\text{neutron absorption in the preceding generation}}$$

Four Factor Formula

A group of fast neutrons produced by fission can enter into several reactions. Some of these reactions reduce the size of the neutron group while other reactions allow the group to increase in size or produce a second generation. There are four factors that are completely independent of the size and shape of the reactor that give the inherent multiplication ability of the fuel and moderator materials without regard to leakage. This four factor formula accurately represents the infinite multiplication factor as shown in the equation below.

$$k_\infty = \varepsilon \ p \ f \ \eta$$

where:

- $\varepsilon$ = fast fission factor
- $p$ = resonance escape probability
- $f$ = thermal utilization factor
- $\eta$ = reproduction factor

Each of these four factors, which are explained in the following subsections, represents a process that adds to or subtracts from the initial neutron group produced in a generation by fission.
Fast Fission Factor, ($\varepsilon$)

The first process that the neutrons of one generation may undergo is fast fission. Fast fission is fission caused by neutrons that are in the fast energy range. Fast fission results in the net increase in the fast neutron population of the reactor core. The cross section for fast fission in uranium-235 or uranium-238 is small; therefore, only a small number of fast neutrons cause fission. The fast neutron population in one generation is therefore increased by a factor called the fast fission factor. The fast fission factor ($\varepsilon$) is defined as the ratio of the net number of fast neutrons produced by all fissions to the number of fast neutrons produced by thermal fissions. The mathematical expression of this ratio is shown below.

$$\varepsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}$$

In order for a neutron to be absorbed by a fuel nucleus as a fast neutron, it must pass close enough to a fuel nucleus while it is a fast neutron. The value of $\varepsilon$ will be affected by the arrangement and concentrations of the fuel and the moderator. The value of $\varepsilon$ is essentially 1.00 for a homogenous reactor where the fuel atoms are surrounded by moderator atoms. However, in a heterogeneous reactor, all the fuel atoms are packed closely together in elements such as pins, rods, or pellets. Neutrons emitted from the fission of one fuel atom have a very good chance of passing near another fuel atom before slowing down significantly. The arrangement of the core elements results in a value of about 1.03 for $\varepsilon$ in most heterogeneous reactors. The value of $\varepsilon$ is not significantly affected by variables such as temperature, pressure, enrichment, or neutron poison concentrations. Poisons are non-fuel materials that easily absorb neutrons and will be discussed in more detail later.

Resonance Escape Probability, ($p$)

After increasing in number as a result of some fast fissions, the neutrons continue to diffuse through the reactor. As the neutrons move they collide with nuclei of fuel and non-fuel material and moderator in the reactor losing part of their energy in each collision and slowing down. While they are slowing down through the resonance region of uranium-238, which extends from about 6 eV to 200 eV, there is a chance that some neutrons will be captured. The probability that a neutron will not be absorbed by a resonance peak is called the resonance escape probability. The resonance escape probability ($p$) is defined as the ratio of the number of neutrons that reach thermal energies to the number of fast neutrons that start to slow down. This ratio is shown below.

$$p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$
The value of the resonance escape probability is determined largely by the fuel-moderator arrangement and the amount of enrichment of uranium-235 (if any is used). To undergo resonance absorption, a neutron must pass close enough to a uranium-238 nucleus to be absorbed while slowing down. In a homogeneous reactor the neutron does its slowing down in the region of the fuel nuclei, and this condition is easily met. This means that a neutron has a high probability of being absorbed by uranium-238 while slowing down; therefore, its escape probability is lower. In a heterogeneous reactor, however, the neutron slows down in the moderator where there are no atoms of uranium-238 present. Therefore, it has a low probability of undergoing resonance absorption, and its escape probability is higher.

The value of the resonance escape probability is not significantly affected by pressure or poison concentration. In water moderated, low uranium-235 enrichment reactors, raising the temperature of the fuel will raise the resonance absorption in uranium-238 due to the doppler effect (an apparent broadening of the normally narrow resonance peaks due to thermal motion of nuclei). The increase in resonance absorption lowers the resonance escape probability, and the fuel temperature coefficient for resonance escape is negative (explained in detail later). The temperature coefficient of resonance escape probability for the moderator temperature is also negative. As water temperature increases, water density decreases. The decrease in water density allows more resonance energy neutrons to enter the fuel and be absorbed. The value of the resonance escape probability is always slightly less than one (normally 0.95 to 0.99).

The product of the fast fission factor and the resonance escape probability (ε p) is the ratio of the number of fast neutrons that survive slowing down (thermalization) compared to the number of fast neutrons originally starting the generation.

**Thermal Utilization Factor, (f)**

Once thermalized, the neutrons continue to diffuse throughout the reactor and are subject to absorption by other materials in the reactor as well as the fuel. The thermal utilization factor describes how effectively thermal neutrons are absorbed by the fuel, or how well they are utilized within the reactor. The *thermal utilization factor* (f) is defined as the ratio of the number of thermal neutrons absorbed in the fuel to the number of thermal neutrons absorbed in any reactor material. This ratio is shown below.

\[
\frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all reactor materials}} = f
\]

The thermal utilization factor will always be less than one because some of the thermal neutrons absorbed within the reactor will be absorbed by atoms of non-fuel materials.
An equation can be developed for the thermal utilization factor in terms of reaction rates as follows.

\[
    f = \frac{\text{rate of absorption of thermal neutrons by the fuel}}{\text{rate of absorption of thermal neutrons by all reactor materials}}
\]

\[
    f = \frac{\Sigma^U_n \phi^U V^U}{\Sigma^U_n \phi^U V^U + \Sigma^m_n \phi^m V^m + \Sigma^p_n \phi^p V^p}
\]

The superscripts U, m, and p refer to uranium, moderator, and poison, respectively. In a heterogeneous reactor, the flux will be different in the fuel region than in the moderator region due to the high absorption rate by the fuel. Also, the volumes of fuel, moderator, and poisons will be different. Although not shown in the above equation, other non-fuel materials, such as core construction materials, may absorb neutrons in a heterogeneous reactor. These other materials are often lumped together with the superscript designation OS, for "other stuff." To be completely accurate, the above equation for the thermal utilization factor should include all neutron-absorbing reactor materials when dealing with heterogeneous reactors. However, for the purposes of this text, the above equation is satisfactory.

In a homogeneous reactor the neutron flux seen by the fuel, moderator, and poisons will be the same. Also, since they are spread throughout the reactor, they all occupy the same volume. This allows the previous equation to be rewritten as shown below.

\[
    f = \frac{\Sigma^U_n}{\Sigma^U_n + \Sigma^m_n + \Sigma^p_n}
\]

Equation (3-1) gives an approximation for a heterogeneous reactor if the fuel and moderator are composed of small elements distributed uniformly throughout the reactor.

Since absorption cross sections vary with temperature, it would appear that the thermal utilization factor would vary with a temperature change. But, substitution of the temperature correction formulas (see Module 2) in the above equation will reveal that all terms change by the same amount, and the ratio remains the same. In heterogeneous water-moderated reactors, there is another important factor. When the temperature rises, the water moderator expands, and a significant amount of it will be forced out of the reactor core. This means that N^m, the number of moderator atoms per cm\(^3\), will be reduced, making it less likely for a neutron to be absorbed by a moderator atom. This reduction in N^m results in an increase in thermal utilization as moderator temperature increases because a neutron now has a better chance of hitting a fuel atom. Because of this effect, the temperature coefficient for the thermal utilization factor is positive. The amount of enrichment of uranium-235 and the poison concentration will affect the thermal utilization factor in a similar manner as can be seen from the equation above.
Example:

Calculate the thermal utilization factor for a homogeneous reactor. The macroscopic absorption cross section of the fuel is 0.3020 cm\(^{-1}\), the macroscopic absorption cross section of the moderator is 0.0104 cm\(^{-1}\), and the macroscopic absorption cross section of the poison is 0.0118 cm\(^{-1}\).

Solution:

\[
\begin{align*}
\eta &= \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_m^U + \Sigma_p^U} \\
&= \frac{0.3020 \text{ cm}^{-1}}{0.3020 \text{ cm}^{-1} + 0.0104 \text{ cm}^{-1} + 0.0118 \text{ cm}^{-1}} \\
&= 0.932
\end{align*}
\]

Reproduction Factor, (\(\eta\))

Most of the neutrons absorbed in the fuel cause fission, but some do not. The reproduction factor (\(\eta\)) is defined as the ratio of the number of fast neutrons produced by thermal fission to the number of thermal neutrons absorbed in the fuel. The reproduction factor is shown below.

\[
\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}
\]

The reproduction factor can also be stated as a ratio of rates as shown below.

\[
\eta = \frac{\text{rate of production of fast neutrons by thermal fission}}{\text{rate of absorption of thermal neutrons by the fuel}}
\]

The rate of production of fast neutrons by thermal fission can be determined by the product of the fission reaction rate (\(\Sigma_a^U \phi^U\)) and the average number of neutrons produced per fission (\(\nu\)). The average number of neutrons released in thermal fission of uranium-235 is 2.42. The rate of absorption of thermal neutrons by the fuel is \(\Sigma_a^U \phi^U\). Substituting these terms into the equation above results in the following equation.

\[
\eta = \frac{\Sigma_a^U \phi^U \nu}{\Sigma_a^U \phi^U}
\]

Table 1 lists values of \(\nu\) and \(\eta\) for fission of several different materials by thermal neutrons and fast neutrons.
### TABLE 1
Average Number of Neutrons Liberated in Fission

<table>
<thead>
<tr>
<th>Fissile Nucleus</th>
<th>Thermal Neutrons</th>
<th>Fast Neutrons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\nu$</td>
<td>$\eta$</td>
</tr>
<tr>
<td>Uranium-233</td>
<td>2.49</td>
<td>2.29</td>
</tr>
<tr>
<td>Uranium-235</td>
<td>2.42</td>
<td>2.07</td>
</tr>
<tr>
<td>Plutonium-239</td>
<td>2.93</td>
<td>2.15</td>
</tr>
</tbody>
</table>

In the case where the fuel contains several fissionable materials, it is necessary to account for each material. In the case of a reactor core containing both uranium-235 and uranium-238, the reproduction factor would be calculated as shown below.

$$\eta = \frac{N_{U^{235}} \sigma_{f^{U^{235}}} \nu^{U^{235}}}{N_{U^{235}} \sigma_a^{U^{235}} + N_{U^{238}} \sigma_a^{U^{238}}} \qquad (3-2)$$

**Example:**

Calculate the reproduction factor for a reactor that uses 10% enriched uranium fuel. The microscopic absorption cross section for uranium-235 is 694 barns. The cross section for uranium-238 is 2.71 barns. The microscopic fission cross section for uranium-235 is 582 barns. The atom density of uranium-235 is $4.83 \times 10^{21}$ atoms/cm$^3$. The atom density of uranium-238 is $4.35 \times 10^{22}$ atoms/cm$^3$. $\nu$ is 2.42.

**Solution:**

Use Equation (3-2) to calculate the reproduction factor.

$$\eta = \frac{N_{U^{235}} \sigma_{f^{U^{235}}} \nu^{U^{235}}}{N_{U^{235}} \sigma_a^{U^{235}} + N_{U^{238}} \sigma_a^{U^{238}}}$$

$$\begin{align*}
\eta &= \left(4.83 \times 10^{21} \text{ atoms/cm}^3\right) \left(582 \times 10^{-24} \text{ cm}^2\right) \left(2.42\right) \\
&= \left(4.83 \times 10^{21} \text{ atoms/cm}^3\right) \left(694 \times 10^{-24} \text{ cm}^2\right) + \left(4.35 \times 10^{22} \text{ atoms/cm}^3\right) \left(2.71 \times 10^{-24} \text{ cm}^2\right) \\
&= 1.96
\end{align*}$$
As temperature varies, each absorption and fission microscopic cross section varies according to the $1/v$ relationship (see Module 2). Since both the numerator and the denominator change equally, the net change in $\eta$ is zero. Therefore, $\eta$ changes only as uranium-235 enrichment changes. $\eta$ increases with enrichment because there is less uranium-238 in the reactor making it more likely that a neutron absorbed in the fuel will be absorbed by uranium-235 and cause fission.

To determine the reproduction factor for a single nuclide rather than for a mixture, the calculation may be further simplified to the one shown below.

$$\eta = \frac{\sigma_f v}{\sigma_a}$$

### Effective Multiplication Factor

The infinite multiplication factor can fully represent only a reactor that is infinitely large, because it assumes that no neutrons leak out of the reactor. To completely describe the neutron life cycle in a real, finite reactor, it is necessary to account for neutrons that leak out. The multiplication factor that takes leakage into account is the effective multiplication factor ($k_{\text{eff}}$), which is defined as the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption and leakage in the preceding generation.

The effective multiplication factor may be expressed mathematically as shown below.

$$k_{\text{eff}} = \frac{\text{neutron production from fission in one generation}}{\text{neutron absorption in the preceding generation} + \text{neutron leakage in the preceding generation}}$$

So, the value of $k_{\text{eff}}$ for a self-sustaining chain reaction of fissions, where the neutron population is neither increasing nor decreasing, is one. The condition where the neutron chain reaction is self-sustaining and the neutron population is neither increasing nor decreasing is referred to as the critical condition and can be expressed by the simple equation $k_{\text{eff}} = 1$.

If the neutron production is greater than the absorption and leakage, the reactor is called supercritical. In a supercritical reactor, $k_{\text{eff}}$ is greater than one, and the neutron flux increases each generation. If, on the other hand, the neutron production is less than the absorption and leakage, the reactor is called subcritical. In a subcritical reactor, $k_{\text{eff}}$ is less than one, and the flux decreases each generation.
When the multiplication factor of a reactor is not equal to exactly one, the neutron flux will change and cause a change in the power level. Therefore, it is essential to know more about how this factor depends upon the contents and construction of the reactor. The balance between production of neutrons and their absorption in the core and leakage out of the core determines the value of the multiplication factor. If the leakage is small enough to be neglected, the multiplication factor depends upon only the balance between production and absorption, and is called the infinite multiplication factor ($k_{\infty}$) since an infinitely large core can have no leakage. When the leakage is included, the factor is called the effective multiplication factor ($k_{\text{eff}}$).

The effective multiplication factor ($k_{\text{eff}}$) for a finite reactor may be expressed mathematically in terms of the infinite multiplication factor and two additional factors which account for neutron leakage as shown below.

$$k_{\text{eff}} = k_{\infty} \cdot \mathcal{L}_{f} \cdot \mathcal{L}_{t}$$

**Fast Non-Leakage Probability ($\mathcal{L}_{f}$)**

In a realistic reactor of finite size, some of the fast neutrons leak out of the boundaries of the reactor core before they begin the slowing down process. The *fast non-leakage probability* ($\mathcal{L}_{f}$) is defined as the ratio of the number of fast neutrons that do not leak from the reactor core to the number of fast neutrons produced by all fissions. This ratio is stated as follows.

$$\mathcal{L}_{f} = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

**Thermal Non-Leakage Probability ($\mathcal{L}_{t}$)**

Neutrons can also leak out of a finite reactor core after they reach thermal energies. The *thermal non-leakage probability* ($\mathcal{L}_{t}$) is defined as the ratio of the number of thermal neutrons that do not leak from the reactor core to the number of neutrons that reach thermal energies. The thermal non-leakage probability is represented by the following.

$$\mathcal{L}_{t} = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

The fast non-leakage probability ($\mathcal{L}_{f}$) and the thermal non-leakage probability ($\mathcal{L}_{t}$) may be combined into one term that gives the fraction of all neutrons that do not leak out of the reactor core. This term is called the *total non-leakage probability* and is given the symbol $\mathcal{L}_{t}$, where $\mathcal{L}_{t} = \mathcal{L}_{f} \cdot \mathcal{L}_{t}$. $\mathcal{L}_{f}$ and $\mathcal{L}_{t}$ are both affected by a change in coolant temperature in a heterogeneous water-cooled, water-moderated reactor. As coolant temperature rises, the coolant expands. The density of the moderator is lower; therefore, neutrons must travel farther while slowing down. This effect increases the probability of leakage and thus decreases the non-leakage probability. Consequently, the temperature coefficient (defined later) for the non-leakage probabilities is negative, because as temperature rises, $\mathcal{L}_{f}$ and $\mathcal{L}_{t}$ decrease.
Six Factor Formula

With the inclusion of these last two factors it is possible to determine the fraction of neutrons that remain after every possible process in a nuclear reactor. The effective multiplication factor \( k_{\text{eff}} \) can then be determined by the product of six terms.

\[
k_{\text{eff}} = \epsilon \mathcal{G}_f \ p \ \mathcal{L}_f \ f \ \eta \tag{3-3}
\]

Equation (3-3) is called the *six factor formula*. Using this six factor formula, it is possible to trace the entire neutron life cycle from production by fission to the initiation of subsequent fissions. Figure 1 illustrates a neutron life cycle with nominal values provided for each of the six factors. Refer to Figure 1 for the remainder of the discussion on the neutron life cycle and sample calculations. The generation begins with 1000 neutrons. This initial number is represented by \( N_0 \). The first process is fast fission and the population has been increased by the neutrons from this fast fission process. From the definition of the fast fission factor it is possible to calculate its value based on the number of neutrons before and after fast fission occur.

\[
\epsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}} = \frac{1040}{1000} = 1.04
\]

The total number of fast neutrons produced by thermal and fast fission is represented by the quantity \( N_0 \epsilon \).

Next, it can be seen that 140 neutrons leak from the core before reaching the thermal energy range. The fast non-leakage probability is calculated from its definition, as shown below.

\[
\mathcal{G}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}} = \frac{1040 - 140}{1040} = 0.865
\]

The number of neutrons that remain in the core during the slowing down process is represented by the quantity \( N_0 \epsilon \mathcal{G}_f \).
Figure 1  Neutron Life Cycle with $k_{ef} = 1$
The next step in the analysis is to consider the number of neutrons that are absorbed in the intermediate energy level. The probability of escaping this resonance absorption (p) is stated as follows.

\[ p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}} \]

\[ = \frac{720}{900} \]

\[ = 0.80 \]

The number of neutrons entering the thermal energy range is now represented by the quantity \( N_0 \times p \).

After reaching thermal energies, 100 neutrons leak from the core. The value for \( \Phi_t \) can be calculated by substitution of the known values in the definition as shown below.

\[ \Phi_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}} \]

\[ = \frac{620}{720} \]

\[ = 0.861 \]

The number of thermal neutrons available for absorption anywhere in the core is represented by the quantity \( N_0 \times \Phi_t \times p \).

Figure 1 indicates that 125 neutrons were absorbed in non-fuel materials. Since a total of 620 thermal neutrons were absorbed, the number absorbed by the fuel equals 620 - 125 = 495. Therefore, the thermal utilization factor can be calculated as follows.

\[ f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in any reactor material}} \]

\[ = \frac{495}{620} \]

\[ = 0.799 \]
The final factor numerically describes the production of fission neutrons resulting from thermal neutrons being absorbed in the fuel. This factor is called the reproduction factor ($\eta$). The value for the reproduction factor can be determined as shown below.

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

$$\eta = \frac{1000}{495} = 2.02$$

The number of fission neutrons that exist at the end of the life cycle which are available to start a new generation and cycle is represented by the quantity $N_o \cdot e \cdot p \cdot f \cdot \eta$.

In the example illustrated in Figure 1, $k_{eff}$ is equal to one. Therefore, 1000 neutrons are available to start the next generation.

Example:

10,000 neutrons exist at the beginning of a generation. The values for each factor of the six factor formula are listed below. Calculate the number of neutrons that exist at the points in the neutron life cycle listed below.

1) Number of neutrons that exist after fast fission.
2) Number of neutrons that start to slow down in the reactor.
3) Number of neutrons that reach thermal energies.
4) Number of thermal neutrons that are absorbed in the reactor.
5) Number of thermal neutrons absorbed in the fuel.
6) Number of neutrons produced from thermal fission.

$$e = 1.031 \quad p_f = 0.889 \quad f = 0.751$$
$$p = 0.803 \quad p_t = 0.905 \quad \eta = 2.012$$

Solution:

1) $N = N_o \cdot e = 10,310$
2) $N = N_o \cdot e \cdot p_f = 9,166$
3) $N = N_o \cdot e \cdot p \cdot p_f = 7,360$
4) $N = N_o \cdot e \cdot p \cdot p_t = 6,661$
5) $N = N_o \cdot e \cdot p \cdot p_f \cdot f = 5,002$
6) $N = N_o \cdot e \cdot p \cdot p_f \cdot f \cdot \eta = 10,065$
Neutron Life Cycle of a Fast Reactor

The neutron life cycle in a fast reactor is markedly different than that for a thermal reactor. In a fast reactor, care is taken during the reactor design to minimize thermalization of neutrons. Virtually all fissions taking place in a fast reactor are caused by fast neutrons. Due to this, many factors that are taken into account by the thermal reactor neutron life cycle are irrelevant to the fast reactor neutron life cycle. The resonance escape probability is not significant because very few neutrons exist at energies where resonance absorption is significant. The thermal non-leakage probability does not exist because the reactor is designed to avoid the thermalization of neutrons. A separate term to deal with fast fission is not necessary because all fission is fast fission and is handled by the reproduction factor.

The thermal utilization factor is modified to describe the utilization of fast neutrons instead of thermal neutrons. The reproduction factor is similarly modified to account for fast fission instead of thermal fission.

Summary

The important information in this chapter is summarized on the following pages.
Neutron Life Cycle Summary

- The infinite multiplication factor, $k_\infty$, is the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption in the preceding generation.

- The effective multiplication factor, $k_{\text{eff}}$, is the ratio of the number of neutrons produced by fission in one generation to the number of neutrons lost through absorption and leakage in the preceding generation.

- Critical is the condition where the neutron chain reaction is self-sustaining and the neutron population is neither increasing nor decreasing.

- Subcritical is the condition in which the neutron population is decreasing each generation.

- Supercritical is the condition in which the neutron population is increasing each generation.

- The six factor formula is stated as $k_{\text{eff}} = \frac{e \, \varphi_f \, p \, \eta}{f}$. Each of the six factors is defined below.

  \[
  e = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}
  \]

  \[
  \varphi_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}
  \]

  \[
  p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}
  \]

  \[
  \varphi_r = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}
  \]

  \[
  f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all reactor materials}}
  \]

  \[
  \eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}
  \]
Neutron Life Cycle Summary (Cont.)

- The thermal utilization factor can be calculated from the macroscopic cross section for absorption of reactor materials using Equation (3-1).

\[
f = \frac{\Sigma^U_a}{\Sigma^U_a + \Sigma^m + \Sigma^p}
\]

- The reproduction factor can be calculated based on the characteristics of the reactor fuel using Equation (3-2).

\[
\eta = \frac{N\ U_{235} \sigma_{a_{235}}^{U_{235}} \nu_{U_{235}}}{N\ U_{235} \sigma_{a_{235}}^{U_{235}} + N\ U_{238} \sigma_{a_{238}}^{U_{238}}}
\]

- The number of neutrons present at any point in the neutron life cycle can be calculated as the product of the number of neutrons present at the start of the generation and all the factors preceding that point in the life cycle.

- The thermal utilization factor is effected by the enrichment of uranium-235, the amount of neutron poisons, and the moderator-to-fuel ratio.

- The reproduction factor is effected by the enrichment of uranium-235.

- The resonance escape probability is effected by the enrichment of uranium-235, the temperature of the fuel, and the temperature of the moderator.

- An increase in moderator temperature will have the following effects.

  - Increase the thermal utilization factor
  - Decrease resonance escape probability
  - Decrease fast non-leakage probability
  - Decrease thermal non-leakage probability